International Real Business Cycles, Portfolio Diversification and Valuation Effects

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Abstract

This paper presents an international real business cycle model in the spirit of Backus, Kehoe, and Kydland (92) with endogenously determined portfolio allocations under incomplete markets. It jointly studies the properties of the portfolio side of the economy, which includes the valuation effects, asset returns and portfolio allocations, along with the more typical international macro variables. The model generates a substantial portfolio home bias, which is dependent on the combination of the consumption home bias parameter and the elasticity of substitution between domestic and foreign traded goods. A higher level of consumption bias determines a higher level of portfolio bias, but the elasticity of substitution must be small in order for the result to hold. The home asset is a good hedge against movements of international prices, which make home physical capital lose value in response to productivity shocks. Using productivity and demand shocks, the model generates also an adequate amount of assets' valuations, which drives the distinction between accounting current account and changes in the net foreign asset position of a country, but does not resolve the Backus-Smith puzzle for standard parameterizations. Adding a further shock to investment absorption does not help in this respect either.

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1 Introduction

As the relationships across countries have become tighter in the last few decades, the degree of trade openness and the domestic holdings of international gross assets have significantly increased and larger shares of foreign assets have progressively found place in the portfolios of the major economies of the world. However, this

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increasing internationalization of goods and financial markets has been characterized by growing global imbalances that are somehow larger than expected. One of the most evident and studied outcomes of this process is the current situation of the United States. Compared to the beginning of the eighties, (the ratio to GDP of) the sum of the American exports and imports has doubled, the foreign assets and liabilities have increased by more than three times, and foreign assets represent about 25% of Americans' invested wealth. However, its current account (CA) has shown a persistent and growing deficit for all that period. Being the CA a proxy of the change of the international net debt position, its deterioration has raised much concern about the long run sustainability of this imbalanced situation and has sparked the recent strand of literature that has provided numerous explanations for the underlying adjustment mechanisms.

Gourinchas and Rey (07), for example, analyze the US international position adjustments in deviation from slow moving trends and show the crucial role played by the valuation effects in the sustainability of its current imbalance. They use a sophisticated elaboration of the intertemporal budget constraint, but their results are not nested in a general equilibrium framework. Lane and Milesi-Ferretti (04) and (07) stress the relevance of the valuation channel at global level as they document the growing holdings of assets and liabilities across countries and their link to exchange rate adjustments. In Obstfeld and Rogoff (05) and Blanchard, Giavazzi, and Sa (05) the real exchange rate is the driving force of the adjustment process; they are the first to consider the imperfect substitutability of international assets as the key element to explain the dynamics of the forces involved in the large American CA deficit.

Although very insightful, all these models call for a general equilibrium approach to the problem. And this is the direction that I want to pursue in this paper proposing a DSGE model for the international real business cycles, in the spirit of Backus, Kehoe, and Kydland (92).

An international RBC model that aims to describe the current situation of the world economy, characterized by large imbalances and remarkable valuation effects, has to endogenously determine also portfolio allocations and assets' valuations consistent with the empirical observation. This model builds on an otherwise standard framework, in which capital is the only required productive factor, and solves for portfolio choices under incomplete markets. It succeeds in replicating both the typically large portfolio home bias and these valuation effects. Furthermore, it provides a theoretical accounting of the role of the RER in the dynamics of the valuation effects and shows that the current international conditions are more compatible with cycles driven by absorption rather than supply side shocks.

Although very convenient, the assumption of complete international financial markets is not yet realistic. The model recognizes the importance of the endogenous determination of investment positions and asset returns in order to determine the macro equilibrium of the economy when financial markets are incomplete. It identifies shocks to productivity and to consumers' preferences as the primary causes of imbalances,

while the valuation channel is explicitly incorporated in my analysis through the portfolio decision of the consumers[1]. The solution of the model is obtained by applying the approximation technique introduced by Devereux and Sutherland (06a) and (06b).

With respect to incomplete markets, the macro and portfolio equilibrium of the economy depend on each other. A standard first order approximation of a model does not deliver a solution to the optimal portfolio allocation, which therefore remains undetermined. However, Devereux and Sutherland show that a suitable combination of a second order approximation of the portfolio conditions and a first order approximation of the macro side of the model can be used to derive the optimal portfolio holdings[2]. The repetition of the same procedure to higher orders delivers a solution for higher order dynamics of the variables[3].

In my simple setup, I assume that there are only two countries and two imperfectly substitutable tradable goods, that labor supply is fixed, and the only productive factor is capital. There are two internationally traded equity assets, one issued by the domestic firm, the other by the foreign firm; the two assets are not perfect substitutes. Similarly to Ferrero, Gertler, and Svensson (07), the model implements a dynamic version of the static model in Obstfeld and Rogoff (05)[4]. Furthermore, it explicitly considers optimal capital and investment decisions, which turn out to be a crucial factor in characterizing the tension between consumption and saving, leading to the observed international distribution of savings.

The model generates a substantial portfolio home bias, which is directly dependent on the combination of the parameter of home bias in consumption and the elasticity of substitution between domestic and foreign traded goods. A higher level of consumption bias determines a higher level of portfolio bias, but the elasticity of substitution must be small in order for the result to hold.

The mechanism generating the portfolio bias is triggered by the physical capital accumulation. Physical investment is assumed to be made in domestic goods[5]. When a positive productivity shock hits the domestic economy, the capital productivity increases and the domestic consumer wants to invest more exactly when the relative price of the domestic good is lower and the term of trade is depreciating. However, the corresponding capital gains and higher returns on the domestic equity asset allow this asset to work as a hedge against the

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[1]Ferrero (07) and Engel and Rogers (06), for example, study those causes, but the same type of shocks are used by Blanchard et al. (05) as well.

[2]Portfolio decisions are based on evaluation of risk, which is missing from a first order approximation. Therefore, a second order approximation is required to solve for the portfolio allocation. They show that only steady state portfolio holdings are required in order to fully solve the first order approximation of the macro side of the model; on the other hand, only first order accurate solutions for the macro variables of the model are necessary to solve for the second order accurate approximation of the portfolio conditions, which involves only second moments of the non-portfolio variables. Simultaneously combining these two elements provides a solution for the portfolio shares.

[3]Tille and van Wincoop (06) implement a numerical approach to the same methodology based on a fixed point search of the optimal portfolio allocation. Evans and Hnatkovska (05) propose a very elegant alternative solution method based on projection and perturbation techniques that allows for heteroskedasticity in the asset returns, but which heavily departs from standard DSGE solution methods.

[4]Production decisions are endogenized, even though the goods market structure is different.

[5]Relaxing this assumption would make the diversification less pronounced, but not null since it would still depend on the consumption bias.

international loss of value of the extra accumulated capital.

As the substitutability of goods increases, the changes in relative prices following a domestic productivity shock have to do less in order to re-equilibrate the goods markets; the domestic assets lose that insuring property that leads to the portfolio bias. Consumers basically have to use the portfolio leverage more heavily to protect themselves against the country specific risk. In this model, it turns out that the home bias degree is not sensitive to the income capital share and the relative variance of the shocks.

I then use the model to asses some typical issues analyzed in the open economy RBC literature. The interesting contribution of the paper is that it can provide further insights about the behavior of assets holdings and assets returns. This kind of model predicts a higher cross-country correlation of consumption than of output levels for reasons related to risk sharing, while data usually suggest the opposite; my model is in line with this standard outcome. The model generates countercyclical net exports and CA measures and procyclical changes of the net asset position. The distinction between actual changes of the net asset position and accounting CA and their different cyclical properties are crucially driven by the valuation effects. This result shows that the model implies valuation effects of adequate size.

The empirical correlation between relative consumption and real exchange rate is usually negative and very close to zero. As Backus and Smith (93) point out, in a theoretical model with complete markets the real exchange rate is proportional to the ratio of the marginal utilities of consumption. The correlation between relative consumption and real exchange rate can only be high and positive. Despite the presence of incomplete markets, however, my benchmark parameterization does not solve this puzzle and suggests that market incompleteness is not the only factor that matters in this respect. Increasing the relative variability of the demand shocks leads the correlation down toward negative values. Following this result, I propose a modification of the model in which investment specific shocks are introduced in order to affect absorption and consumption, while moving domestic prices up. The modification does not help to solve the puzzle in this model.

Heatcote and Perri (08) use the same type of open economy RBC model introducing non-diversifiable labor income risk. They find that, even with complete markets, a large degree of home bias in portfolio allocation can be obtained as a normal effect of risk sharing due to the negative correlation between labor income and domestic returns in response to domestic productivity shocks. Their result depends on the insuring properties of the response of international relative prices to changes in relative income and on the presence of capital and investment choices in their model. Kollmann (06) presents a numerical technique to solve for portfolio holdings in a model with complete markets and consumption home bias. He uses his model to assess the portfolio home bias and the business cycle properties of a measure of current account that includes valuation effects. Coeurdacier, Kollmann, and Martin (09) propose a complete market model

with a two-shock two-asset structure, in which both stocks and bonds can be traded and investment shocks are added to the standard TFP shocks. They show that bonds can be used to hedge the risk due to unfavorable responses of the term of trade to productivity shocks, while equity assets are used to hedge the negative comovements between wages and dividends caused by shocks that generate movements of the relative investment orthogonal to the term of trade.

Caballero, Farhi, and Gourinchas (08) shift their focus to the role of global financial markets and in particular the role of regional asset supplies in the determination of capital flows. Their general equilibrium model, aside from the persistent CA deficit of the US region, generates endogenously low interest rates and portfolio allocations consistent with data. However, they abandon the assumption of long run reversibility of the current account and take the structure of assets supplied across regions as exogenous.

The paper is organized as follows. Section 2 describes the model in detail, part of the algebra necessary to make the model suitable to apply the solution procedure is left to Appendix A. Section 3 presents the first order log linearization of the model and derives the approximation of portfolio conditions necessary to apply Devereux and Sutherland method. Appendix C briefly summarizes the second order approximation of the model. Section 4 shows the results and Section 5 concludes.

2   The model

I consider a two-region model in which the two countries (Home and Foreign) are assumed to have the same size. I assume complete markets inside each country, but incomplete international financial markets. This is equivalent to assuming that there is only one representative consumer in each economy. The two countries trade in international securities in order to hedge against aggregate country idiosyncratic shocks. One innovation is a productivity shock, the other is a shock to preferences affecting the intertemporal discount factor. Given market incompleteness, an endogenous discount factor that responds in a negative way to the aggregate consumption level is used in order to ensure the existence of a well defined steady state for net wealth and the other variables of the model.

The shocks to productivity introduce a differential in the rate of growth of productivity across regions; the shocks to preferences can be used to represent shifts in demand due, for instance, to aging of population. These are generally believed to be the causes of the currently observed international imbalances. Each country produces a single tradable good, which can be either consumed or invested in domestic capital, or exported to the other country. Assuming that investment is possible only in domestic goods makes imports motivated only by consumption.

I assume that the consumer has some degree of home bias in consumption. He sets the asset holdings

for the current period based on the expected correlation of their returns with the endogenous consumption-based kernel. Firms set the demand of capital and investment maximizing the discounted expected future dividends that are completely distributed to the shareholders. A simplifying assumption is that each firm is owned only by the domestic representative agent and the domestic discount factor is used to evaluate future dividends.

There are two internationally traded equity assets that represent claims on the dividends of the firm in each country. Prices are flexible and the law of one price holds. However, the home bias allows the real exchange rate to differ from 1. Under these assumptions, the prices' dynamics is fully summarized by the term of trade.

2.1   The Firm

In each country, a representative competitive firm produces a single tradable good using capital as a production factor. I will focus on the home country H, the foreign country is assumed to have the same characteristics if not otherwise specified.

Labor is assumed to be inelastically supplied by consumers and it is fixed to the constant value of 1. A standard Cobb-Douglas production function in this case can be re-adapted to a linear production function in capital only

$$Y_{H,t} = A_t K_t^\sigma$$

where A_t is a productivity shock following an exogenous stochastic process and K_t is the amount of (standardized by labor) capital used by the firm. The parameter $\sigma \in (0,1)$ implies decreasing returns to scale and has the usual interpretation of share of capital income in the production. The firm issues equity to finance its production and maximizes the cash flows it pays in terms of dividends to its stockholders. Equity shares represent only claims on future dividends, while the property of the firm is assigned to the domestic agent by assumption; for this reason, the relevant intertemporal discount factor for the firm is that of the domestic consumer. The firm accumulates the capital and makes the investment decision at each period; it takes the sale price of output on the domestic and foreign market as given.

Let $P_{H,t}$ be the price of the H good in the H market and $P_{H,t}^*$ the corresponding price in the F market expressed in the foreign currency; the equivalent prices for the F good are $P_{F,t}$ and $P_{F,t}^*$. A superscript star indicates foreign variables. Perfect exchange rate pass through is assumed; this implies that the law of one price (LOP) for the traded goods holds. Defining S_t, the nominal exchange rate between the two countries, as the price of currency F in terms of currency H, so that an increase of S is a depreciation of currency H,

LOP implies that

$$P_{H,t} = S_t P^*_{H,t}$$

$$P_{F,t} = S_t P^*_{F,t}$$

Good H can be consumed either at home or abroad; it can alternatively be invested in capital formation, but I assume only domestically. This assumption implies that only domestic goods can be capitalized by domestic firms, while exports and imports are limited to consumption purposes. Once optimal investment has been set, consumers can use the international financial assets to finance imports and enhance consumption if wanted and feasible.

The production function for the foreign country firm is equivalent

$$Y^*_{F,t} = A^*_t (K^*_{j,t})^{\sigma}$$

The productivity shocks are assumed to follow an autoregressive process and are allowed to have some degree of contemporaneous correlation across countries. Backus et al. (92) provide a reference for the calibration of this process.

$$\begin{bmatrix} \log A_t \\ \log A^*_t \end{bmatrix} = \begin{bmatrix} \rho_A & 0 \\ 0 & \rho_{A^*} \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A^*_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{A,t} \\ \varepsilon_{A^*,t} \end{bmatrix} \tag{1}$$

The vector of innovations $\varepsilon'_t = [\begin{array}{cc} \varepsilon_{A,t} & \varepsilon_{A^*,t} \end{array}]$ is i.i.d. with mean zero and constant covariance matrix

$$\begin{bmatrix} v & \\ v_A & v \end{bmatrix}$$

The variance is the same for the two countries. A contemporaneous spillover to the other country (v_A) and different persistence of the effects of the innovations $(\rho_A$ and $\rho_{A^*})$ are allowed.

Nominal dividends are

$$Q_t = P_{H,t} Y_{H,t} - P_{H,t} I_t - G(I_t)$$

where $G(\cdot)$ is the nominal adjustment cost the firm faces for the installation of new capital and I_t represents

the investment chosen for period t. In real terms it becomes

$$q_t = \frac{P_{H,t}}{P_t} (Y_{H,t} - I_t) - g(I_t) \tag{2}$$

where $g(\cdot)$ is the real adjustment cost of new investment and P_t is the CPI price level prevailing in country H and defined in the consumer's section. We define $g(\cdot)$ in terms of units of consumption of the aggregate domestic good C_t, this implies that $G(I_t) = P_t g(I_t)$. It is convenient to interpret this adjustment cost as an installation cost that the firm pays to the consumer as a compensation for the work required to install the new capital.[6] It is also convenient to assume that $g(I_{ss}) = g'(I_{ss}) = 0$, with I_{ss} the steady state level of investment. The second derivative of the function is positive and will have to be calibrated in order to return the right dynamic properties of investment $g''(I_{ss}) = \phi > 0$.

The real dividends maximization problem setup is

$$\max_{K_t} E_t \sum_{i=0}^{\infty} m_{t+i} q_{t+i} \tag{3}$$

$$s.t. \ K_{t+1} = (1 - \delta) K_t + I_t \tag{4}$$

$$Y_{H,t} = A_t K_t^{\sigma} \tag{5}$$

Capital follows the standard law of motion in (4), where δ is the depreciation rate of capital, m_{t+i} is the domestic consumer consumption based discount factor.[7] Substituting the constraints (4) and (5) into (3) and taking the derivative w.r.t. K_{t+1}, we get the following first order condition

$$\frac{P_{H,t}}{P_t} + g'(I_t) = E_t \left\{ m_{t+1} \left[\begin{array}{c} \frac{P_{H,t+1}}{P_{t+1}} \left(\sigma A_{t+1} K_{t+1}^{\sigma-1} + 1 - \delta \right) \\ + (1 - \delta) g'(I_{t+1}) \end{array} \right] \right\} \tag{6}$$

Equation (6), along with the capital accumulation law (4), implicitly determines the optimal solution path of capital and investment.

The ratio $\frac{P_{H,t+1}}{P_{t+1}}$ is endogenously determined in equilibrium at time $t+1$ in order to guarantee that international markets clear and the general equilibrium of the model. The current optimal real dividends depend only on the optimal choice of capital for the next period, since current capital is given and the realization of the productivity shock is known at the time of maximization, the choice of K_{t+1} defines I_t and, hence, the dividends.

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[6] The cost enters the consumer's budget constraint as a source of income and makes the definition of the GDP equation and of Δw slightly easier and more familiar.

[7] m_t is fully defined below.

Given the definition of P_t in (9), the ratio $\frac{P_{H,t}}{P_t}$ can be rewritten as

$$\frac{P_{H,t}}{P_t} = \frac{1}{\left[\lambda + (1-\lambda)\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}} \tag{7}$$

where the term of trade τ_t is defined as

$$\tau_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{F,t}^*}{P_{H,t}^*} \tag{8}$$

2.2 The Consumer

The representative agent consumes a composite good C_t defined by a CES function over the home and foreign good. All goods are tradable, but not perfect substitute, and the elasticity of substitution is given by the parameter θ

$$C_t = \left[\lambda^{\frac{1}{\theta}}(C_{H,t})^{\frac{\theta-1}{\theta}} + (1-\lambda)^{\frac{1}{\theta}}(C_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where $\lambda \in (\frac{1}{2}, 1)$ represents the home bias in consumption (assumed exogenously given). $C_{H,t}$ and $C_{F,t}$ are the H consumer's consumption of the good produced in the H country and of the good produced in the F country respectively. The corresponding for the F consumer is

$$C_t^* = \left[(1-\lambda)^{\frac{1}{\theta}}(C_{H,t}^*)^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}}(C_{F,t}^*)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

The assumption about the home bias introduces an asymmetry in the definition of the consumption bundle that makes the CPIs of the two countries differ, even though the law of price holds. Those price indices P_t and P_t^* are

$$P_t = \left[\lambda P_{H,t}^{1-\theta} + (1-\lambda)P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{9}$$

$$P_t^* = \left[(1-\lambda)(P_{H,t}^*)^{1-\theta} + \lambda(P_{F,t}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

The PPP (purchasing power parity) does not hold and the real exchange rate J_t does not necessarily have to be equal to 1 any more

$$J_t = \frac{S_t P_t^*}{P_t} \tag{10}$$

J_t is defined as the price of the F good in terms of the H good; a decrease of J_t corresponds to an appreciation of the H good/H RER (F good is becoming cheaper), while an increase of J_t corresponds to a depreciation.

The last elements coming from the intraperiod cost minimization problem of the consumer are the relative

demand of H and F goods (here reported for the H consumer[8])

$$C_{H,t} = \lambda \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t$$

$$C_{F,t} = (1 - \lambda) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t \tag{11}$$

Since investment goods can be produced only domestically, $C_{F,t}$ and $C_{H,t}^*$ are respectively the imports and exports of the H country. The flows of resources necessary to satisfy the demand for investment in a country with a shortage of saving have exclusively a financial nature.

Preferences of the consumer are defined over consumption and take the log form

$$E_0 \sum_{t=0}^{\infty} \gamma_t \log C_t$$

The discount factor γ_t endogenously responds to the level of consumption to ensure a stationary steady state distribution of wealth, since with incomplete markets the endogenous variables, in particular the wealth process, would be non stationary making any approximation method potentially imprecise

$$\gamma_t = \beta_t \gamma_{t-1} \tag{12}$$

γ_t is recursively defined starting from the uniperiodal time varying discount factor β_t, which is a decreasing function of the aggregate consumption C_{t-1} of the previous period, taken as exogenous by the consumer. Similarly to Ferrero et al. (07), I assume the following functional form for β_t

$$\beta_t = \frac{\beta_{ss} e^{\zeta_t}}{1 + \psi_1 (\log C_{t-1} - \log C_{ss})} \tag{13}$$

where ζ_t is an autoregressive process such that

$$\zeta_t = \rho_\zeta \zeta_{t-1} + u_{\zeta t} \tag{14}$$

$u_{\zeta t}$ i.i.d. $N(0, v_\zeta)$ and the parameters ψ_1 should be small and positive in order to ensure the desired inverse relation between β and consumption, but still having a smooth transition around the steady state.

Savings can be allocated between two internationally traded equity assets. Those assets represent a claim on the firms' future stream of profits and can obviously generate capital gains and losses as their prices

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[8]For the F consumer the demand functions are symmetric: $C_{H,t}^* = (1 - \lambda) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\theta} C_t^*$ and $C_{F,t}^* = \lambda \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^*$

change over time. The payoffs are closely determined by the properties of the national productivity shocks.

Let $V_{H,t}$ be the share of the H-firm equity that the H consumer chooses to hold from the beginning of period t to the beginning of $t+1$; $V_{F,t}$ is the H consumer's share of the F-firm equity. The analogous for the F consumer would be $V_{H,t}^*$ and $V_{F,t}^*$.

Taking into account the payment of the adjustment costs from the firm to the consumer, the budget constraint of the consumer in nominal terms is

$$(Z_{H,t} + Q_t)V_{H,t-1} + S_t(Z_{F,t}^* + Q_t^*)V_{F,t-1} + G(I_t) = Z_{H,t}V_{H,t} + S_t Z_{F,t}^* V_{F,t} + P_t C_t \tag{15}$$

where $Z_{H,t}$ and $Z_{F,t}^*$ are the prices of 1 share of the H and F indices expressed in local currencies, Q_t are the nominal profits defined above, S_t is the nominal exchange rate.

We can rewrite the budget constraint (15) in terms of gross nominal asset returns $R_{H,t}$ and $R_{F,t}$

$$R_{H,t}(Z_{H,t-1}V_{H,t-1}) + R_{F,t}(S_{t-1}Z_{F,t-1}^* V_{F,t-1}) + G(I_t) = Z_{H,t}V_{H,t} + S_t Z_{F,t}^* V_{F,t} + P_t C_t \tag{16}$$

In order to apply the Devereux-Sutherland solution method, we must express the budget constraint in terms of real net wealth, where the term "net" refers to the difference between domestic holdings abroad and foreign holdings at home.

If we standardize the total supply of equity to 1, the simplest way to express net wealth is to recognize that (16) can be equivalently re-formulated as if the domestic agent holds the whole domestic equity and chooses the share $V_{H,t-1}$ of returns he wants to retain.

After a few simple steps, shown in Appendix A, and defining the net nominal wealth W_t as the sum of the nominal assets holdings $\alpha_{j,t}^{nm}$ for $j = H, F$

$$W_t = \sum_{j=H,F} \alpha_{j,t}^{nm} = -Z_{H,t}(1 - V_{H,t}) + S_t Z_{F,t}^* V_{F,t}$$

the constraint in (16) becomes

$$W_t = R_{H,t}\alpha_{H,t-1}^{nm} + R_{F,t}\alpha_{F,t-1}^{nm} - P_t C_t + Q_t + G(I_t) \tag{17}$$

The final step is to express everything in real terms, relative to the aggregate consumption good in country H, and in terms of the excess returns of the F asset relative to the H asset

$$w_t = r_{x,t}\alpha_{F,t-1} + r_{H,t}w_{t-1} - C_t + q_t + g(I_t) \tag{18}$$

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where, after dividing both sides of (17) by P_t, w_t is the real net wealth and $r_{x,t} = (r_{F,t} - r_{H,t})$ is the real excess return of the F equity over the H equity; $r_{H,t}$ and $r_{F,t}$ are the real counterparts of $R_{H,t}$ and $R_{F,t}$, and $\alpha_{j,t}$ and q_t are the real counterparts of $\alpha_{j,t}^{nm}$ and Q_t.

Keeping the H good as a numeraire for returns and asset prices (and hence wealth as well), and measuring the other components of the constraint in terms of F aggregate goods, the equivalent constraint for the F consumer is given by

$$\frac{1}{J_t} w_t^* = \frac{1}{J_t} \left[r_{x,t} \alpha_{F,t-1}^* + r_{H,t} w_{t-1}^* \right] - C_t^* + q_t^* + g^* \left(I_t \right)$$

From (10), it is easy to see how the real exchange rate J_t is related to the term of trades τ_t

$$J_t = \frac{\left[(1 - \lambda) + \lambda \tau_t^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[\lambda + (1 - \lambda) \tau_t^{1-\theta} \right]^{\frac{1}{1-\theta}}} \tag{19}$$

2.2.1 First Order Conditions

The *f.o.c.* for the consumer maximization problem are more easily derived formulating the problem in the following way

$$\max_{C_t} E_0 \sum_{t=0}^{\infty} \gamma_t \log C_t$$

$$\begin{aligned} s.t. \quad & q_t + g(I_t) + r_{H,t} \alpha_{H,t-1} + r_{F,t} \alpha_{F,t-1} \\ = \quad & \alpha_{H,t} + \alpha_{F,t} + C_t \end{aligned}$$

and subject to (12)-(14), which define the endogenous discount factor γ_t. Taking the derivatives with respect to the $\alpha_{j,t}$ for given prices, we have the standard set of first order conditions

$$E_t \left[m_{t+1} r_{H,t+1} \right] = 1 \tag{20}$$

$$E_t \left[m_{t+1} r_{F,t+1} \right] = 1 \tag{21}$$

where m_{t+1} is the consumption based discount factor defined as $m_{t+1} = \beta_t \frac{C_t}{C_{t+1}}$. Condition (20) and (21) can be combined to obtain the optimality condition in terms of the excess returns $r_{x,t}$

$$E_t \left[m_{t+1} \left(r_{F,t+1} - r_{H,t+1} \right) \right] = E_t \left[m_{t+1} r_{x,t+1} \right] = 0 \tag{22}$$

A transversality condition is associated to the two conditions for $j = H, F$

$$\lim_{T \to \infty} E_t \left[m_{t+T} \alpha_{j,t+T} \right] = 0 \qquad (23)$$

Since the supply of equity assets is assumed to be given exogenously and to be constant, (23) implies that equity prices must not be affected by bubbles.

A similar set of conditions hold for the F consumer, with the real exchange rate J_t appearing in the f.o.c.[9]

$$E_t \left[m_{t+1}^* \frac{J_t}{J_{t+1}} r_{H,t+1} \right] = 1$$

$$E_t \left[m_{t+1}^* \frac{J_t}{J_{t+1}} r_{F,t+1} \right] = 1$$

2.2.2 Market Clearing

The wealth shares of the two countries are complementary to each other. By assumption, the domestic consumer owns the whole domestic fund and decides the share to retain for himself and the share to pass to the foreign agent. The clearing condition simply is

$$\alpha_{F,t} = -\alpha_{F,t}^*$$

$$-\alpha_{H,t} = \alpha_{H,t}^*$$

By construction, the same condition must hold for net wealths as well

$$w_t = -w_t^*$$

We can interpret $\alpha_{F,t}$ as country H total foreign assets, or equivalently as F country foreign liabilities.[10] On the other hand $\alpha_{H,t}^*$ are country H liabilities (country F assets).

Starting from the consumer budget constraint (50) and using the definition of the nominal $alfas^{nm}$ in (51) and of nominal profits $Q_t = P_{H,t} (Y_{H,t} - I_t) - G(I_t)$, we can construct an expression for the change in

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[9] It might be useful to express $\frac{J_t}{J_{t+1}}$ in terms of the term of trade as

$$\frac{J_t}{J_{t+1}} = \frac{\left[(1-\lambda) + \lambda \tau_t^{1-\theta} \right]^{\frac{1}{1-\theta}} \left[\lambda + (1-\lambda)\tau_{t+1}^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[\lambda + (1-\lambda)\tau_t^{1-\theta} \right]^{\frac{1}{1-\theta}} \left[(1-\lambda) + \lambda \tau_{t+1}^{1-\theta} \right]^{\frac{1}{1-\theta}}}$$

[10] These are not simply the H consumer's foreign assets, they account for all the country foreign assets because of the structure of the model.

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the net foreign asset position and relate it to the national accounting definition of current account

$$P_{H,t}Y_{H,t} + R_{H,t}\alpha^{nm}_{H,t-1} + R_{F,t}\alpha^{nm}_{F,t-1} = \alpha^{nm}_{H,t} + \alpha^{nm}_{F,t} + P_t C_t + P_{H,t}I_t$$

$$\Delta\alpha^{nm}_{H,t} + \Delta\alpha^{nm}_{F,t} = (R_{H,t} - 1)\alpha^{nm}_{H,t-1} + (R_{F,t} - 1)\alpha^{nm}_{F,t-1} + P_{H,t}(Y_{H,t} - I_t) - P_t C_t$$

where $P_{H,t}Y_{H,t} - P_t C_t$ can be seen as the equivalent of savings in this model. In real terms

$$\Delta w_t = \Delta\alpha_{H,t} + \Delta\alpha_{F,t} = (r_{H,t} - 1)\alpha_{H,t-1} + (r_{F,t} - 1)\alpha_{F,t-1} + \frac{Y_{H,t} - I_t}{\left[\lambda + (1 - \lambda)\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}} - C_t \qquad (24)$$

This is not exactly the current account reported in national accounting because it includes the valuation effects that are not in the official accounting. The real rates $r_{H,t}$ and $r_{F,t}$ incorporate the capital gain terms $x_{H,t} = \frac{\Delta z_{H,t}}{z_{H,t-1}}$ and $x_{F,t} = \frac{\Delta(J_t z^*_{F,t})}{J_{t-1}z^*_{F,t-1}}$ which are disregarded in the definition of ca_t

$$ca_t = \Delta\alpha_{H,t} + \Delta\alpha_{F,t} - x_{H,t}\alpha_{H,t-1} - x_{F,t}\alpha_{F,t-1} \qquad (25)$$

The equilibrium solution of the asset prices allows to construct the valuation rates and the accounting ca_t.

From the basic accounting equation of GDP it must be that

$$P_{H,t}Y_{H,t} = P_t C_t + P_{H,t}I_t + \left(P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}\right) \qquad (26)$$

$$P_{H,t}Y_{H,t} = P_t C_t + P_{H,t}I_t + \left(P_{H,t}C^*_{H,t} - P_{F,t}C_{F,t}\right) \qquad (27)$$

One important assumption of this model is that imports and exports from one country to the other are done only for consumption reasons; this explains the last two terms on the RHS of (26), where exports are $P_{H,t}C^*_{H,t}$ and imports are $P_{F,t}C_{F,t}$. The definition of net exports NX_t is

$$nx_t = \frac{NX_t}{P_t} = \frac{Y_{H,t} - I_t}{\left[\lambda + (1 - \lambda)\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}} - C_t = \frac{C^*_{H,t} - \tau_t C_{F,t}}{\left[\lambda + (1 - \lambda)\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}$$

Symmetric conditions apply to country F

$$ca_t = -ca^*_t$$

$$\Delta w_t = \Delta\alpha_{H,t} + \Delta\alpha_{F,t} = -\left(\Delta\alpha^*_{H,t} + \Delta\alpha^*_{F,t}\right) = -\Delta w^*_t$$

$$nx_t^* = -\frac{nx_t}{\Upsilon_t} = \frac{\tau_t C_{F,t} - C_{H,t}^*}{\left[(1-\lambda) + \lambda\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}$$

3 Log Linear Model

3.1 Steady State Relations

In steady state the PPP must hold. This implies $\Upsilon_{ss} = \tau_{ss} = 1$, $P_{ss} = P_{H,ss} = P$ and $P_{ss} = S_{ss}P_{ss}^*$. Given the assumption on the exogenous productivity process in (??), $A_{ss} = A_{ss}^* = 1$, also which implies $Y_{H,ss} = K_{ss}$ and $Y_{H,ss}^* = K_{ss}^*$.

Relative demands of goods in (??) and (11), evaluated in steady state, return $C_{H,ss} = \lambda C_{ss}$ and $C_{F,ss} = (1-\lambda)C_{ss}$; from the law of motion of capital (??) $I_{ss} = \delta K_{ss}$. The same relations hold for the F country: $C_{H,ss}^* = (1-\lambda)C_{ss}^*$ and $C_{F,ss}^* = (1-\lambda)C_{ss}^*$; $I_{ss}^* = \delta K_{ss}^*$.

The model is approximated around a zero net wealth position $w_{ss} = w_{ss}^* = 0$ that also implies, from the definitions of ca and nx, that $nx_{ss} = ca_{ss} = 0$. Then $C_{F,ss} = C_{H,ss}^*$ and $C_{ss} = C_{H,ss} + C_{F,ss} = C_{H,ss} + C_{H,ss}^*$, and for the same reason $C_{ss}^* = C_{F,ss}^* + C_{F,ss}$. Using these relations and those just above for $C_{H,ss}^*$ and $C_{F,ss}^*$, we have that $C_{ss} = C_{H,ss} + C_{H,ss}^* = \lambda C_{ss} + (1-\lambda)C_{ss}^* \rightarrow C_{ss} = C_{ss}^*$. From the GDP equation (26)

$$Y_{H,ss} = C_{ss} + I_{ss}$$

$$C_{ss} = Y_{H,ss} - \delta\left(Y_{H,ss}\right)^{\frac{1}{\sigma}} = \left(K_{ss}\right)^\sigma - \delta K_{ss}$$

and for F

$$C_{ss}^* = Y_{F,ss}^* - \delta K_{ss}^* = \left(K_{ss}^*\right)^\sigma - \delta K_{ss}^*$$

the condition $C_{ss} = C_{ss}^*$ implies that in a symmetric equilibrium $Y_{H,ss} = Y_{F,ss}^*$ if, as assumed here, δ is the same across countries.

From the first order conditions of the consumer we get the steady state value of the interest rates, $m_{ss} = \beta_{ss}$ implies $r_{H,ss} = r_{F,ss} = \frac{1}{\beta_{ss}}$. Condition (6) defines the relation between K_{ss} and the structural parameters σ, β_{ss} and δ: $\frac{1}{\beta_{ss}} = \sigma\left(K_{ss}\right)^{\sigma-1} + 1 - \delta$. The choice of the parameters determines the steady state value of capital. Finally, optimal real profits (2) in steady state are $q_{ss} = Y_{H,ss} - \delta K_{ss} = C_{ss}$.

We can see that the steady state financial returns, $r_{H,ss}$ and $r_{F,ss}$, are equal to the marginal productivity of capital corrected by the depreciation rate of capital. We can further solve K_{ss} as a function of the underlying parameters

$$K_{ss} = \left[\frac{1 - \beta_{ss}(1-\delta)}{\sigma\beta_{ss}}\right]^{\frac{1}{\sigma-1}} \tag{28}$$

This allows to express other variables in s.s. as a function of those parameters. In particular $Y_{H,ss}$ and C_{ss} are

$$Y_{H,ss} = \left[\frac{1 - \beta_{ss}(1 - \delta)}{\sigma \beta_{ss}} \right]^{\frac{\sigma}{\sigma - 1}}$$

$$C_{ss} = \left[\frac{1 - \beta_{ss}(1 - \delta(1 - \sigma))}{\sigma \beta_{ss}} \right] \left[\frac{1 - \beta_{ss}(1 - \delta)}{\sigma \beta_{ss}} \right]^{\frac{1}{\sigma - 1}}$$

3.2 Linearized System

In the model there are two symmetric sources of exogenous shocks in each country, one to productivity and another to preferences. Their log linear form is already given by (1) and (14) and is reported here again

$$\hat{a}_t = \rho_A \hat{a}_{t-1} + v\varepsilon_{A,t} + v_{A^*}\varepsilon_{A^*,t}$$

$$\hat{a}_t^* = \rho_{A^*} \hat{a}_{t-1}^* + v\varepsilon_{A^*,t} + v_A\varepsilon_{A,t}$$

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t}$$

$$\hat{\zeta}_t^* = \rho_\zeta \hat{\zeta}_{t-1}^* + \varepsilon_{\zeta^*,t}$$

Where small letter variables represent the log version of the same capital letter variables, for example $a_t = \log A_t$, and $\zeta_t = \log e^{\zeta_t}$. A hat ^ over a variable indicates the log-deviation of a (capital) variable from its steady state. Since all these shocks in levels have a s.s. value of 1, $a_{ss} = \zeta_{ss} = 0$ and the deviation, in this case, just corresponds to the (log of the) shock itself.

The endogenous discount factor (13) depends on the preferences shock and, negatively, on the previous period consumption level, which guarantees the stationarity of the model with incomplete markets. A higher aggregate consumption relative to the steady state level reduces β and decreases individual future consumption relative to today, inducing higher future savings to face higher debt today

$$\hat{\beta}_t = \hat{\zeta}_t - \psi_1 \hat{c}_{t-1}$$

$$\hat{\beta}_t^* = \hat{\zeta}_t^* - \psi_1 \hat{c}_{t-1}^*$$

I turn now to the first order conditions of the consumer. We have the typical Euler equations for consumption in the two countries, for country F the real exchange rate $\hat{j}_t$ is necessary to convert $\hat{c}_t^*$ into

home good terms since the returns are expressed in unit of H good

$$\hat{c}_t = E_t\hat{c}_{t+1} - E_t\hat{r}_{H,t+1} - \hat{\beta}_t$$

$$\hat{c}_t^* + \hat{j}_t = E_t\hat{c}_{t+1}^* + E_t\hat{j}_{t+1} - E_t\hat{r}_{H,t+1} - \hat{\beta}_t^*$$

From (19) $\hat{j}_t = (2\lambda - 1)\hat{\tau}_t$. In this model the whole price dynamic is summarized by the behavior of the term of trade $\hat{\tau}_t$; if $\lambda = 0.5$ the real exchange rate never deviates from its s.s. value of 1 since any deviation from the PPP is due only to consumption bias in this symmetric framework with LOP.

We can decompose the equity returns into a capital gain component, given by the variation of prices, and a flow component, given by the payment of dividends to the stock holders

$$\hat{r}_{H,t+1} = \beta_{ss}\hat{z}_{H,t+1} + (1 - \beta_{ss})\hat{q}_{t+1} - \hat{z}_{H,t}$$

$$\hat{r}_{F,t+1} = \beta_{ss}\hat{z}_{F,t+1}^* + (1 - \beta_{ss})\hat{q}_{t+1}^* - \hat{z}_{F,t}^* + (2\lambda - 1)\hat{\tau}_{t+1} - (2\lambda - 1)\hat{\tau}_t \qquad (33)$$

We can substitute for $\hat{j}_t$, $\hat{r}_{H,t+1}$ and the discount factor into (32)

$$(1 - \psi_1)\hat{c}_t = E_t\hat{c}_{t+1} - \beta_{ss}E_t\hat{z}_{H,t+1} - (1 - \beta_{ss})E_t\hat{q}_{t+1} + \hat{z}_{H,t} - \hat{\zeta}_t$$

$$(1 - \psi_1)\hat{c}_t^* + (2\lambda - 1)\hat{\tau}_t = E_t\hat{c}_{t+1}^* + (2\lambda - 1)E_t\hat{\tau}_{t+1} - \beta_{ss}E_t\hat{z}_{H,t+1} - (1 - \beta_{ss})E_t\hat{q}_{t+1} + \hat{z}_{H,t} - \hat{\zeta}_t^*$$

Then condition (22) becomes

$$E_t(\hat{r}_{x,t+1}) = \beta_{ss}\hat{z}_{F,t+1}^* + (1 - \beta_{ss})E_t\hat{q}_{t+1}^* + (2\lambda - 1)E_t\hat{\tau}_{t+1} - \beta_{ss}E_t\hat{z}_{H,t+1}$$

$$-(1 - \beta_{ss})E_t\hat{q}_{t+1} - \hat{z}_{F,t}^* - (2\lambda - 1)\hat{\tau}_t + \hat{z}_{H,t} = 0$$

The excess return $\hat{r}_{x,t+1}$ is introduced in the system as well because very helpful to apply the DS solution for the steady state investment positions

$$\hat{r}_{x,t} = \beta_{ss}\hat{z}_{F,t}^* + (1 - \beta_{ss})\hat{q}_t^* + (2\lambda - 1)\hat{\tau}_t - \beta_{ss}\hat{z}_{H,t} - (1 - \beta_{ss})\hat{q}_t - \hat{z}_{F,t-1}^* - (2\lambda - 1)\hat{\tau}_{t-1} + \hat{z}_{H,t-1}$$

From the firm's optimal decision of investment, optimal profits (2) can be approximated by

$$\psi_c\hat{q}_t = \hat{y}_{H,t} - \psi_i\hat{\imath}_t - (1 - \lambda)\psi_c\hat{\tau}_t$$

$$\psi_c\hat{q}_t^* = \hat{y}_{F,t}^* - \psi_i\hat{\imath}_t^* + (1 - \lambda)\psi_c\hat{\tau}_t$$

real profits depend positively on domestic output, negatively on investment which represents a cost, and they depend on the relative prices, represented by the term of trade (this is the open economy channel). The adjustment cost in (2) does not appear in the linearization because we have assumed it has a null first derivative in steady state. The coefficients ψ_c and ψ_i are defined from the steady state relations and corresponds to the ratios to GDP of investment and consumption. They are

$$\psi_i = \frac{I_{ss}}{Y_{H,ss}} = \frac{\sigma \beta_{ss} \delta}{1 - \beta_{ss}(1 - \delta)} = \frac{I_{ss}^*}{Y_{H,ss}^*}$$

$$\psi_c = \frac{C_{ss}}{Y_{H,ss}} = \frac{1 - \beta_{ss}[1 - \delta(1 - \sigma)]}{1 - \beta_{ss}(1 - \delta)} = \frac{C_{ss}^*}{Y_{H,ss}^*}$$

The log linear version of aggregate production and of the aggregate capital law of motion are standard, for the H country we have

$$\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta \hat{\imath}_t$$

$$\hat{y}_{H,t} = \hat{a}_t + \sigma \hat{k}_t$$

and symmetric relations hold for the F country.

The last two sets of equations of the system come from the cash flows maximization first order condition of the firms (6) and the market clearing conditions, given by the GDP equations (26) and (27) in real terms.

The first order condition of the maximization of dividends (6), which links the expected capital productivity to the consumption path and the term of trade, and its equivalent version for the F country reduce to

$$[1 - \beta_{ss}(1 - \delta)] \left[E_t \hat{a}_{t+1} + (\sigma - 1) E_t \hat{k}_{t+1} \right] - \beta_{ss}(1 - \delta) \phi I_{ss} E_t \hat{\imath}_{t+1} =$$
$$- (1 - \psi_1) \hat{c}_t - \hat{\zeta}_t + E_t \hat{c}_{t+1} + (1 - \lambda) [E_t \hat{\tau}_{t+1} - \hat{\tau}_t] - \phi I_{ss} \hat{\imath}_t$$
$$[1 - \beta_{ss}(1 - \delta)] \left[E_t \hat{a}_{t+1}^* + (\sigma - 1) E_t \hat{k}_{t+1}^* \right] - \beta_{ss}(1 - \delta) \phi I_{ss} E_t \hat{\imath}_{t+1}^* =$$
$$- (1 - \psi_1) \hat{c}_t^* - \hat{\zeta}_t^* + E_t \hat{c}_{t+1}^* - (1 - \lambda) [E_t \hat{\tau}_{t+1} - \hat{\tau}_t] - \phi I_{ss} \hat{\imath}_t^*$$

The GDP equation incorporates the external trade channel as a buffer that guarantees the international equilibrium between supply and demand in the two goods markets. The approximation is standard, linking total domestic production to the total level of consumption at home and in the foreign country, the domestic

investment, and the relative competitiveness represented by the term of trade. At a first step

$$\hat{y}_{H,t} = \psi_c \hat{c}_t + \psi_i \hat{\imath}_t + (1-\lambda)\psi_c \hat{c}^*_{H,t} - (1-\lambda)\psi_c \hat{c}_{F,t}$$

$$\hat{y}^*_{F,t} = \psi_c \hat{c}^*_t + \psi_i \hat{\imath}^*_t + (1-\lambda)\psi_c \hat{c}_{F,t} - (1-\lambda)\psi_c \hat{c}^*_{H,t}$$

the approximation of the relative demand equation (11) is given by $\hat{c}_{F,t}$ (and equivalently $\hat{c}^*_{H,t}$ for the foreign good)

$$\hat{c}_{F,t} = \hat{c}_t - \lambda\theta\hat{\tau}_t$$

$$\hat{c}^*_{H,t} = \hat{c}^*_t + \lambda\theta\hat{\tau}_t$$

we can, finally, substitues these two into the the GDP equation to obtain

$$\hat{y}_{H,t} = \psi_c \left[\lambda\hat{c}_t + (1-\lambda)\hat{c}^*_t\right] + \psi_i \hat{\imath}_t + 2\lambda(1-\lambda)\psi_c\theta\hat{\tau}_t$$

$$\hat{y}^*_{F,t} = \psi_c \left[\lambda\hat{c}^*_t + (1-\lambda)\hat{c}_t\right] + \psi_i \hat{\imath}^*_t - 2\lambda(1-\lambda)\psi_c\theta\hat{\tau}_t$$

The second order approximation of the model, necessary to solve for the dynamics of the portfolio holdings, is left to Appendix 2.C.

3.2.1 DS approximations

A set of crucial approximations are required to apply the solution method developed by Devereaux and Sutherland (06a) to the model. The most important is the linearization of the consumer's budget constraint. Since wealth and excess returns need to be approximated around a steady state value of zero, we use a Taylor expansion for the linearization, adjusting it to the usual log linear approximation where feasible.

So, once we replace the definition of real profits in (18) in order to eliminate $g(I_t)$ we have

$$w_t = r_{x,t}\alpha_{F,t-1} + r_{H,t-1}w_{t-1} - C_t + \frac{Y_{H,t} - I_t}{\left[\lambda + (1-\lambda)\tau_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}$$

The appropriate linearization of (18) is then

$$\hat{w}_t = \hat{r}_{x,t}\tilde{\alpha}_{F,ss} + \frac{1}{\beta_{ss}}\hat{w}_{t-1} - \psi_c \hat{c}_t + \hat{y}_{H,t} - \psi_i \hat{\imath}_t - (1-\lambda)(1-\psi_i)\hat{\tau}_t$$

where $\tilde{\alpha}_{F,ss} = \frac{\alpha_{F,ss}}{\beta_{ss}Y_{H,ss}}$ and $\hat{w}_t = \frac{w_t - w_{ss}}{Y_{H,ss}} = \frac{w_t}{Y_{H,ss}}$. So we will have a description of the evolution of the net wealth relative to domestic country income (in real terms). As explained by Devereux and Sutherland

(06a) the term $\hat{r}_{x,t}\tilde{\alpha}_{F,ss}$ behaves as an i.i.d. shock which can be exogenized (ε_t in the system).

In order to actually get the portfolio holdings in steady state, we need to derive the second order approximation of the portfolio equations. Following the same steps as in the paper of Devereux and Sutherland leads to the conditions for the two countries

$$E_t \left[\hat{r}_{x,t+1} + \frac{1}{2}\hat{r}^2_{x,t+1} - \hat{c}_{t+1}\hat{r}_{x,t+1} \right] = 0 \tag{42}$$

$$E_t \left[\hat{r}_{x,t+1} + \frac{1}{2}\hat{r}^2_{x,t+1} - \hat{c}^*_{t+1}\hat{r}_{t+1} - \hat{\jmath}_{t+1}\hat{r}_{x,t+1} \right] = 0 \tag{43}$$

where $\hat{r}_{x,t+1}$ and $\hat{r}^2_{x,t+1}$ are defined as

$$\hat{r}_{x,t+1} = \left(\hat{r}_{F,t+1} - \hat{r}_{H,t+1} \right)$$

$$\hat{r}^2_{x,t+1} = \left(\hat{r}^2_{F,t+1} - \hat{r}^2_{H,t+1} \right)$$

Combining equations (42) and (43), we get the second moment condition used to derive the solution for the portfolio holdings

$$E_t \left[\left(\hat{c}_{t+1} - \hat{c}^*_{t+1} - (2\lambda - 1)\hat{\tau}_{t+1} \right) \hat{r}_{x,t+1} \right] = 0 \tag{44}$$

Taking the approximation of the same portfolio first order conditions at the third order and combining them allows to derive the solution for the dynamic behavior of asset allocations. So the corresponding of (42) and (43) at the third order are

$$E_t \left[\begin{array}{c} \hat{r}_{x,t+1} + \frac{1}{2}\hat{r}^2_{x,t+1} + \frac{1}{6}\hat{r}^3_{x,t+1} \\ -\hat{c}_{t+1}\hat{r}_{x,t+1} - \frac{1}{2}\hat{c}_{t+1}\hat{r}^2_{x,t+1} + \frac{1}{2}\hat{c}^2_{t+1}\hat{r}_{x,t+1} \end{array} \right] = 0 \tag{45}$$

$$E_t \left[\begin{array}{c} \hat{r}_{x,t+1} + \frac{1}{2}\hat{r}^2_{x,t+1} + \frac{1}{6}\hat{r}^3_{x,t+1} \\ -\hat{c}^*_{t+1}\hat{r}_{x,t+1} - \frac{1}{2}\hat{c}^*_{t+1}\hat{r}^2_{x,t+1} + \frac{1}{2}\hat{c}^{*2}_{t+1}\hat{r}_{x,t+1} \\ - (2\lambda - 1)\hat{\tau}_{t+1}\hat{r}_{x,t+1} - (2\lambda - 1)\hat{\tau}_{t+1}\hat{r}^2_{x,t+1} \\ + \frac{1}{2}(2\lambda - 1)^2 \hat{\tau}^2_{t+1}\hat{r}_{x,t+1} + \frac{1}{2}(2\lambda - 1)\hat{c}^*_{t+1}\hat{\tau}_{t+1}\hat{r}_{x,t+1} \end{array} \right] = 0 \tag{46}$$

Now subtracting (46) from (45) delivers the moment condition

$$E_t \begin{bmatrix} -\hat{r}_{x,t+1} \left(\hat{c}_{t+1} - \hat{c}^*_{t+1} - (2\lambda - 1)\,\hat{\tau}_{t+1} \right) \\ -\frac{1}{2}\hat{r}^2_{x,t+1} \left(\hat{c}_{t+1} - \hat{c}^*_{t+1} - (2\lambda - 1)\,\hat{\tau}_{t+1} \right) \\ +\frac{1}{2}\hat{r}_{x,t+1} \left(\hat{c}^2_{t+1} - \hat{c}^{*2}_{t+1} - (2\lambda - 1)^2\,\hat{\tau}^2_{t+1} - (2\lambda - 1)\,\hat{c}^*_{t+1}\hat{\tau}_{t+1} \right) \end{bmatrix} = 0$$

Adding (45) and (46) provides the second condition necessary to apply the solution method

$$E_t \hat{r}_{x,t+1} = \begin{bmatrix} -\frac{1}{2}\hat{r}^2_{t+1} - \frac{1}{6}\hat{r}^3_{x,t+1} + \frac{1}{2}\hat{r}_{x,t+1} \left(\hat{c}_{t+1} + \hat{c}^*_{t+1} + (2\lambda - 1)\,\hat{\tau}_{t+1} \right) \\ +\frac{1}{4}\hat{r}^2_{x,t+1} \left(\hat{c}_{t+1} + \hat{c}^*_{t+1} + (2\lambda - 1)\,\hat{\tau}_{t+1} \right) \\ -\frac{1}{4}\hat{r}_{x,t+1} \left(\hat{c}^2_{t+1} + \hat{c}^{*2}_{t+1} + (2\lambda - 1)^2\,\hat{\tau}^2_{t+1} + (2\lambda - 1)\,\hat{c}^*_{t+1}\hat{\tau}_{t+1} \right) \end{bmatrix}$$

4 Calibration and Results

4.1 Calibration

The calibration of the model is made in two stages. In the first stage I exploit the fact that the steady state solution of the portfolio allocation does not depend on the variance of the shocks. The first order solution for the excess returns depends only on the productivity shocks, but not on the demand shocks, so given the particular form of the moment condition in (44) variances simply drop off. This property is very convenient because it allows for setting some important parameters for given relative variance of the shocks, looking at the implications of these parameters for the steady state portfolio home bias degree generated by the model. The last parameters are then set in the second stage of the calibration in an attempt to match the empirical moments of some other variables. For this reason the variances of the innovations are all the same and just equal to 1 here; there are no spillovers and the autoregressive coefficients are set to .9.

The values for the capital share of income $\sigma = .35$, the discount factor $\beta_{ss} = .99$ and the capital depreciation rate $\delta = .025$ are very standard and come from Backus et al. (92). Assuming that one period in the model corresponds to a quarter these values imply that the annual discount factor is .96 and the annual depreciation rate is 10%. They also determine a steady state consumption to GDP ratio $\psi_c = 0.75$ and investment to GDP ratio $\psi_i = 0.25$, which are in line with U.S. data if government expenditure is not included in GDP. The consumption home bias parameter λ is chosen to be 0.8 which returns an import to GDP steady state ratio of 15%, ψ_1 defines the speed at which the net wealth reverts to its steady state value after a shock, it must be positive and small so that this assumption of stationarity does not affect the short run dynamics of the model, I pick $\psi_1 = .01$. Finally the elasticity of substitution between H and F goods θ is .7. Table 1 summarizes the choice of these parameters.

This calibration produces a large portfolio home bias: $V_H = 90.2\%$ and by definition $V_F = 9.8\%$. However,

Calibration	
Capital share	$\sigma = .35$
Discount factor	$\beta_{ss} = .99$
Capital depreciation	$\delta = .025$
Consumption home bias	$\lambda = .8$
Goods subst. elasticity	$\theta = .7$
β responsiveness to cons.	$\psi_1 = .01$
Shocks parameters:	
	$v = 1 = v_\zeta$
	$\rho_A = \rho_{A^*} = .9 = \rho_\zeta$
	$v_A = .2$

Table 1: Calibration: First Stage - Matching steady state values

the choice of a small elasticity of substitution is crucial in order for this result to hold. In this benchmark calibration θ has been set to .7, this is smaller than what it is usually preferred in literature and falls into the lower range of the empirical estimates. However, this is a typical feature of this type of models and the fact that it must be small has a particular meaning for the interaction that θ has with the capital accumulation in generating the portfolio bias, I will discuss this point looking at the impulse responses functions of the model below. Figure 1 shows how the portfolio bias changes when θ varies, V_H remains below 1 basically for $\theta < .9$, then it quickly increases over 1 meaning that the home consumer is going short in the foreign assets. For higher values of elasticity we would get the opposite result of foreign portfolio bias; the threshold value of θ for the switch depends on λ. Figure 2 shows, as expected, that the degree of portfolio home bias is increasing in the degree of consumption bias.

I calibrate the variances of the innovations and the other parameters entering the shocks processes in order to match the moments reported in Table 2, where $cor(.)$ indicates the correlation between two variables and $sd\{\frac{a}{b}\}$ indicates the ratio of the standard deviations of variable a and b. The moments generated by the model are compared to the empirical counterparts found by Backus et al. (92) for a panel of countries and to some empirical data for the U.S. covering the sample 1970-2007, in which a broad definition of rest of the world has been used[11].

The variance covariance matrix of the innovations is

$$
\begin{bmatrix}
v & v_A & 0 & 0 \\
v_A & v & 0 & 0 \\
0 & 0 & v_\zeta & 0 \\
0 & 0 & 0 & v_\zeta
\end{bmatrix}
=
\begin{bmatrix}
.09^2 & .0016 & 0 & 0 \\
.0016 & .09^2 & 0 & 0 \\
0 & 0 & .015^2 & 0 \\
0 & 0 & 0 & .015^2
\end{bmatrix}
$$

The persistency of the productivity shocks is assumed to be slightly larger than that of the consumption

[11] These data come from a dataset I have been constructing for another working paper I'm working on.

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shock, $\rho_A = \rho_{A^*} = .85$ and $\rho_\zeta = \rho_{\zeta^*} = .7$. Finally ϕ, the parameter representing the curvature of the convex adjustment cost, is chosen to be 0.001.[12] As in many other theoretical models where some degree of consumption sharing is in place, matching the correlation of outputs across countries returns a higher correlation of relative consumptions, while in data the opposite is often observed.

4.2 Results

First of all, I evaluate the implications of the model under this benchmark calibration for the moments of net exports nx_t, the accounting definition of current account ca_t and the change in the net wealth Δw_t (that is the change in the net foreign assets position of the domestic country), which are relevant variables in open economy. Then I plot the impulse responses functions to the shocks of the model and use them to describe the hedging mechanism revealed by the model and to make some considerations about assets valuation.

4.2.1 International RBC moments

Even though the model underperforms in generating enough variability relative to output for the three measures of international balance reported in the first three rows of Table 3, the correlations with output expressing the cyclical properties of the three are quite interesting. It is well known, as reported by Kollman (06) for example, that while the national accounting measures of current account are strongly countercyclical, as soon as valuation effects are accounted for in measuring the gross assets and liabilities positions, the variations of the net assets turn out to be mostly acyclical or slightly procyclical. The large difference between ca and Δw due to valuation effects has been stressed by Gourinchas and Rey (07) and a good international RBC model should aim to replicate this difference. In my model the size of the valuation effects is large enough to make the sign of the correlation flip from $-.42$ for the CA to $.12$ for the net assets variations.

Recalling equation (25), we can derive a first order approximated expression of the CA which reads

$$\hat{ca}_t = \Delta \hat{w}_t + \beta_{ss}\tilde{\alpha}_{F,ss}\left[\left(\hat{z}_{H,t} - \hat{z}_{H,t-1}\right) - \left(\hat{z}^*_{F,t} - \hat{z}^*_{F,t-1}\right) - (2\lambda - 1)\left(\hat{\tau}_t - \hat{\tau}_{t-1}\right)\right]$$

where $\hat{ca}_t$ is the deviation of the CA from its steady state value of zero relative to steady state GDP $\hat{ca}_t = \frac{CA_t}{Y_{H,ss}}$, so defined in the same fashion as wealth.

The last raw of Table 3 shows the correlation between relative consumption and the real exchange rate J_t, data consistently tell that it is small and close to zero, often negative, but in any model with complete markets this correlation is always positive and large, because the real exchange rate is proportional to the

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[12] This value implies a very small adjustment cost. The rest of the calibration doesn't seem particularly sensitive to it, for ϕ ranging between 0.1 and 0.001.

Moment	Model	Data	BKK
$sd\left[\frac{c}{y}\right]$	.58	$.55 \sim .6$	.5
$sd\left[\frac{i}{y}\right]$	2.7	$2.1 \sim 3.4$	3
$cor\,(c,y)$	.79	$.65 \sim .85$	.75
$cor\,(i,y)$	.91	$.7 \sim .8$	.9
$cor\,(c,c^*)$	.66	.6	$.1 \sim .3$
$cor\,(y,y^*)$	.36	.8	$.3 \sim .5$

Table 2: Calibration: Second Stage - Matching empirical moments. sd[a/b] is the ratio of the standard deviations of variables a and b. cor( ) is the correlation between two variables.

ratio of marginal utilities of the two countries. Backus and Smith (93) pointed out this discrepancy and incomplete markets usually mitigate it, but despite the presence of a second type of shocks to preferences in this model the correlation is still very high (.76).

A productivity shocks to domestic production creates a relative higher supply of domestic good, its price then becomes cheaper and the term of trade defined as $\tau = \frac{P_F}{P_H}$ increases (τ devaluates) while relative home consumption is higher in response to the higher productivity at home. On the other hand demand side shocks induce an opposite correlation, since the higher relative demand for the domestic good (due to the consumption home bias) causes an increase of P_H, that is a valuation of τ. If the relative magnitude of the demand innovations is not large enough, as in our benchmark parameterization, the first effect would still dominate the second. Figure 3 shows that setting the relative variances of the two shocks equal to 0.6 ($v_\zeta = .6v$) would already take care of the Backus-Smith puzzle. This result would suggest to modify the baseline model including other absorption type shocks, as for example specific-investment shocks. I would talk about it in Section 4.2.4, but we will see they are not helping much in solving the problem.

4.2.2 The underlying forces

Productivity and demand shocks are commonly believed to be the sources of the observed American current account dynamics[13], in particular American productivity has been higher than the rest of the world productivity during the 90s and a large increase in the current consumption level of American people has been associated to factors such as aging population or the will of holding American assets (in particular government debt) by Asian central banks. So Figure 4 and 5 present the impulse responses functions of consumption, investment and assets returns to the two types of shocks in the model.

The first set of responses in Figure 4 help to understand the mechanism behind the portfolio home bias. When a productivity shock hits the home economy, the home investment increases and an extra accumulation of capital occurs exactly in correspondence of a depreciation of the real exchange rate. Since

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[13]See for example Ferrero (07) and Blanchard et al. (05).

Moment	Model	Data	Others
$sd\left[\frac{nx}{y}\right]$	.06	$2 \sim 3.5$	$.5 \sim 1.5^a$
$sd\left[\frac{ca}{y}\right]$	.04	.7	
$sd\left[\frac{\Delta w}{y}\right]$	.17	1.3	
$cor\,(nx, y)$	$-.41$	$-.38$	$-.3^a$
$cor\,(ca, y)$	$-.42$	$-.23$	$< 0^b$
$cor\,(\Delta w, y)$	.12	.008	$\simeq 0^b$
$cor\,(c/c^*, J)$	.76	.06	$< 0, \simeq 0^c$

Table 3: Relative variances and correlation for some important open economy variables. a Backis et al. (92), b Kollman (06), c Backus-Smith (93). sd[a/b] is the ratio of the standard deviations of variables a and b. cor( ) is the correlation between two variables.

capital investment is made in domestic goods, if the elasticity of substitution of traded goods θ is small the swing of RER will be large and also the relative loss of value of the accumulated capital will be large.

The home asset provides a good hedge against this devaluation of the domestic capital, as we can see from the second plot of the panel, because it pays a higher capital gain (at net of the RER valuation effect) than the foreign asset. In terms of the returns shown in the last plot of the panel, we can see the productivity shock causes a positive excess return. We have seen, talking about the Backus-Smith puzzle above, that consumption and RER are conditionally very well correlated in response to a productivity shock. The foreign asset, whose returns are mainly determined by the response of the RER, has to offer relatively higher returns to the home agent in order to compensate for this positive correlation.

Figure 5 shows the impulse responses functions corresponding to a negative demand shock, in this model this is associated to a decrease of the intertemporal discount factor β_t and hence to an increase of current consumption level, which is the case we are interested in. We can notice that there is no excess return in response to this type of shock, as I said to justify the calibration procedure I followed. Comparing the first plot of the two figures, we have a confirmation of why the demand shock is not able to correct for the Backus-Smith puzzle. Its conditional negative correlation between consumption and RER is not strong enough to counterbalance the conditional positive correlation following a productivity shock.

A useful feature of the model is that it allows making some direct considerations about the assets valuation. I first look at the decomposition of the returns based on the approximation in (33) of the following two equations which define the real returns in terms of the domestic good

$$r_{H,t} = 1 + \frac{\Delta z_{H,t}}{z_{H,t-1}} + \frac{q_t}{z_{H,t-1}}$$

$$r_{F,t} = \frac{J_t}{J_{t-1}}\left(1 + \frac{\Delta z^*_{F,t}}{z^*_{F,t-1}} + \frac{q^*_t}{z^*_{F,t-1}}\right)$$

The foreign asset returns present two possible sources of capital gain: one is the change of the price of the equity itself in term of foreign goods $z_{F,t}^*$, the second is the depreciation (or appreciation) of the real exchange rate J_t. Figure 6 shows the decomposition of the returns after a shock to home and foreign productivity. The model attributes a large share of the total returns determination to the valuation channel of the RER, roughly speaking about 50 to 60% of the returns' response can be explained by it. It would be interesting to compare this prediction of the model to its empirical counterpart.

In Figure 7 I look again at the difference between ca_t and Δw_t, as done at the beginning of this section, to get a sense of the valuation effects implied by the RER movements. The figure plots the responses of these two variables and the RER to a home productivity shock and a (negative) demand shock. The figure is quite significant because it shows that the currently observed situation for the U.S., where the strong depreciation of the RER has created favorable valuation effects and has made the changes of the net position typically smaller than the CA, has more likely been produced by demand type shocks. Combining the two shocks after assuming a larger magnitude for the demand shock would replicate the real case, so also this result calls for the introduction of an extra absorption type shock.

4.2.3   The Dynamics of Portfolio Holdings

It's possible to apply the same solution method used to find the steady state portfolio allocations to derive a solution for the dynamic responses of the asset holdings. The model must be approximated up to the second order, while the portfolio conditions up to the third. Those conditions are derived in Section 2.3, while the approximation of the macro side of the model is reported in Appendix C. Figure 8 shows the responses of the assets holdings to our two types of shocks.

First of all, this must be taken as a qualitative exercise since the magnitude of the responses is implausibly large. Corsetti, Dedola and LeDuc (08) empirical study of the international effects of US productivity and demand shocks finds that the responses to shocks of the portfolio holdings in their VAR can be larger in magnitude than the responses of other variables such as consumption or output, but usually by no more than a factor of 10^2. Here the responses to the demand shock are in line with this empirical result, but those to the production shock are way too large (they are about 10^4 times the other responses). A remedy for this excessively strong reaction of the solution to shocks could probably be the introduction of some kind of friction to trade.

However, we can see from Figure 8 is that after a home productivity shock the domestic consumer will reduce his holdings of foreign assets $\hat{\alpha}_{F,t}$ trying to switch to domestic assets. $\hat{\alpha}_{H,t} = \hat{w}_t - \hat{\alpha}_{F,t}$, which represents the negative of his foreign liabilities, increases and so also his liabilities decrease, but less than the foreign assets and the net wealth is temporarily positive. The home consumer responds to the productivity

shock reinforcing his own portfolio bias, then the symmetric structure of the model induce the foreign consumer to behave in the same way.

An increase in domestic consumption due to a demand shock produces a relative higher demand for the home good because of the consumption bias of the home consumer and so higher profits for the domestic firm. The foreign agent responds trying to increase his holdings of home assets, so $\hat{\alpha}_{H,t}^{*} = -\hat{\alpha}_{H,t}$ goes up, but this implies the home agent has to reduce his holdings of domestic assets for a while reallocating part of his portfolio toward foreign assets in order to leave some room for the liabilities to grow and as a matter of facts the response of $\hat{\alpha}_{F,t}$ is positive. The home consumer finds very convenient this portfolio reallocation since he would finance his higher consumption basically borrowing from the foreign country. Both the shocks generate positive comovements of assets and liabilities.

4.2.4 Adding Investment Specific Shocks

In the benchmark version of the model, the firm obtains the investment good using home consumption goods according to a simple "one to one" transformation technology, the price of the investment good and the price of the home consumption good must be the same ($P_{H,t}$). An investment-specific productivity shock allows the relative price of the two types of goods to differ from 1. A positive shock makes the transformation of consumption goods into capital goods more efficient, less inputs are required to produce the same amount of investment good, which is equivalent to say that investment in terms of consumption gets cheaper.

The firm's problem defined in Section 2.1 is slightly modified by the introduction of the investment transformation technology. Let I_t' be the investment measured in efficiency units and keep the same notation I_t as above for the consumption good used as input to produce I_t'. The transformation technology is $I_t' = e^{\eta_t} I_t$, where η_t is the investment shock assumed to follow a first order autoregressive process characterized by i.i.d. $N(0, v_\eta)$ innovations $\varepsilon_{\eta,t}$.

The relative price of the investment good in terms of consumption good is just the inverse of the productivity shock $e^{-\eta_t}$. Only the capital accumulation law and the firm's first order condition are affected by this modification in the following way

$$K_{t+1} = (1-\delta) K_t + I_t' = (1-\delta) K_t + e^{\eta_t} I_t$$

$$\frac{P_{H,t}}{P_t} e^{-\eta_t} + g'(I_t') = E_t \left\{ m_{t+1} \left[\begin{array}{c} \frac{P_{H,t+1}}{P_{t+1}} \left(\sigma A_{t+1} K_{t+1}^{\sigma-1} + (1-\delta) e^{-\eta_{t+1}} \right) \\ + (1-\delta) g'(I_{t+1}') \end{array} \right] \right\}$$

Greenwood, Hercowitz and Krusell (97), and many others after them, show the relevance of these shocks

in determining business cycles fluctuations. Raffo (08) shows that investment-specific shocks could explain the quantity prices correlation puzzle using a model with GHH preferences[14], in which labor supply strongly reacts to the investment shocks and international absorption is split over both investment and consumption. This shock increases the demand and the price of the domestic good, which can be associated to higher consumption if the market supply of labor increases enough. This cannot happen here.

In my model this type of shock increases investment and domestic prices on impact as expected for an absorption type shock, but consumption is crowded out by the higher demand for capital goods in a way similar to that of a close economy model, since investment is made with domestic goods. Once production picks up pushing prices down and consumption back to higher levels, a positive conditional correlation between consumption differentials and RER is introduced which does not help in solving the puzzle. Figure 9 shows the impulse responses to this shock in a baseline parameterization in which I have set the variance of the investment shock equal half of the variance of the productivity shock.

5   Conclusions

This paper implements a DSGE model with consumers portfolio choice under incomplete markets to study the properties and behavior of the portfolio side of the economy which includes the valuation effects, asset returns, portfolio allocations, jointly with the more typical international macro variables such as the current account, consumption and saving decisions, the real exchange rate.

Two crucial features of the current world economy are large imbalances and remarkable valuation effects. A satisfactory international RBC model has to endogenously determines portfolio allocations and assets' valuations consistent with those observed in the data. This model succeds in achieving both these tasks. The model also shows that a large portion of the dynamics of the valuation effects is actually explained by movements of the RER and that the current international conditions are more compatible with cycles driven by absorption than supply side shocks.

Two assets representing claims on the dividends of the national firm of each country are freely traded, but perfect risk sharing is not attained and a high degree of portfolio home bias is generated by the model. The presence of capital accumulation in domestic goods and consumption home bias is enough to rationalize this bias; the home equity asset provides insurance to the home agent against the international devaluation of the domestic capital in response to a productivity shock. Heathcote and Perri (08) use also non-diversifiable labor income risk negatively correlated with domestic returns to obtain the same result, but investment is actually all one needs to generate the home bias. The result hinges on a low elasticity of substitution

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[14]Introduced by Greenwood, Hercowitz and Huffman (1998): "Investment, Capacity Utilization and the Real Business Cycle", *AER.*

between domestic and foreign traded goods in the lower end of the empirical estimates, which is becoming a popular feature of many international RBC models. Coeurdacier et al. (08) show that this dependence on the elasticity of substitution can be avoided introducing bonds hedging the term of trade risk, the home bias in the equity portfolio allocation is generated at this point by the necessity of hedging the negative comovements of wages and dividends that are orthogonal to the term of trade responses to shocks.

Once a standard calibration is adopted in order to match the relative variances of output, consumption and investment, the solution for the portfolio holdings allows the model to endogenously generate those valuations effects of the assets positions which have drawn much attention in the recent debate about the sustainability of the international imbalances. In particular, I focus on the distinction between accounting current account definition and its real counterpart, which is the change in the net foreign asset position of a country. While the CA is countercyclical, the change in net assets is slightly procyclical, as observed in the data. The model also allows a theoretical decomposition of the relative importance of pure capital gains and the real exchange rate channel in the response of returns to productivity shocks - we find that the RER accounts for 50-60% of the total response.

The Backus and Smith puzzle still arises and it turns out that it is mainly due to the low relative volatility of the demand shocks implied by the calibration of the model with respect to the productivity shocks. This and the low relative volatility of the CA and of the change in net foreign assets suggest a more active role for demand type shocks. Using investment specific shocks, as done by others, does not achieve the scope in my case, because consumption is crowded out by the higher demand for capital goods in a way similar to that of close economy models. So I think further steps in this direction are necessary to improve this model, along with obtaining more specific empirical counterparts of the impulse responses functions presented in this paper.

Appendix

A   Derivation of the consumer's Budget Constraint

We can start from (15), which is reported here for convenience

$$(Z_{H,t} + Q_t)V_{H,t-1} + S_t(Z^*_{F,t} + Q^*_t)V_{F,t-1} + G(I_t) \tag{47}$$

$$= Z_{H,t}V_{H,t} + S_t Z^*_{F,t}V_{F,t} + P_t C_t$$

We can rewrite the budget constraint taking into account explicitly the capital gains and valuations components of the assets, the second and third term of the LHS of (47) becomes

$$\frac{(Z_{H,t} + Q_t)}{Z_{H,t-1}} Z_{H,t-1}V_{H,t-1} + \frac{S_t(Z^*_{F,t} + Q^*_t)}{S_{t-1}Z^*_{F,t-1}} S_{t-1} Z^*_{F,t-1}V_{F,t-1}$$

$$(1 + \frac{\Delta Z_{H,t}}{Z_{H,t-1}} + \frac{Q_t}{Z_{H,t-1}})Z_{H,t-1}V_{H,t-1} + \frac{S_t}{S_{t-1}}(1 + \frac{\Delta Z^*_{F,t}}{Z^*_{F,t-1}} + \frac{Q_t}{Z^*_{F,t-1}})S_{t-1} Z^*_{F,t-1}V_{F,t-1}$$

Then using (??) in (47) and expressing the gross nominal returns of assets as[15]

$$R_{H,t} = (1 + R^{net}_{H,t}) = (1 + \frac{\Delta Z_{H,t}}{Z_{H,t-1}} + \frac{Q_t}{Z_{H,t-1}}) \tag{48}$$

$$R_{F,t} = (1 + R^{net}_{F,t}) = \frac{S_t}{S_{t-1}}(1 + \frac{\Delta Z^*_{F,t}}{Z^*_{F,t-1}} + \frac{Q^*_t}{Z^*_{F,t-1}})$$

we can rewrite the budget constraint as

$$R_{H,t}(Z_{H,t-1}V_{H,t-1}) + R_{F,t}(S_{t-1}Z^*_{F,t-1}V_{F,t-1}) + G(I_t)$$

$$= Z_{H,t}V_{H,t} + S_t Z^*_{F,t}V_{F,t} + P_t C_t \tag{49}$$

In order to obtain (17) a few simple manipulations are required. So adding and subtracting $(Z_{H,t} + Q_t)$ on the LHS of the constraint, which corresponds to the value of the index at time t plus the dividens it pays, and rearranging the terms, and applying (48), we can replace the decision of $V_{H,t-1}$ with that of $-(1-V_{H,t-1})$ which is the share of the index passed on to the foreign consumer and which would be a negative term in

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[15]It should be clear from this notation that $R^{net}_{H,t}$ and $R^{net}_{F,t}$ are the net asset returns.

computing the H consumer net wealth.

Let's modify (49) hence

$$(Z_{H,t} + Q_t) - R_{H,t}Z_{H,t-1}(1 - V_{H,t-1}) + R_{F,t}(S_{t-1}Z^*_{F,t-1}V_{F,t-1}) + G(I_t)$$

$$= Z_{H,t}V_{H,t} + S_t Z^*_{F,t}V_{F,t} + P_t C_t$$

or moving $Z_{H,t}$ on the LHS

$$Q_t + G(I_t) - R_{H,t}Z_{H,t-1}(1 - V_{H,t-1}) + R_{F,t}(S_{t-1}Z^*_{F,t-1}V_{F,t-1}) +$$

$$= -Z_{H,t}(1 - V_{H,t}) + S_t Z^*_{F,t}V_{F,t} + P_t C_t \tag{50}$$

Define now the nominal asset holdings $\alpha^{nm}_{j,t}$ for $j = H, F$ as

$$\alpha^{nm}_{H,t} = -Z_{H,t}(1 - V_{H,t})$$

$$\alpha^{nm}_{F,t} = S_t Z^*_{F,t}V_{F,t} \tag{51}$$

The net nominal wealth t is then the current value of the sum of the holdings of assets at net of income and after the consumption choice has been taken

$$W_t = \sum_{j=H,F} \alpha^{nm}_{j,t} = -Z_{H,t}(1 - V_{H,t}) + S_t Z^*_{F,t}V_{F,t}$$

In our specific case, the constraint (50) becomes

$$W_t = R_{H,t}\alpha^{nm}_{H,t-1} + R_{F,t}\alpha^{nm}_{F,t-1} - P_t C_t + Q_t + G(I_t) \tag{52}$$

The final step is to express everything in real terms relative to the aggregate consumption good in country H and in terms of the excess returns. Deviding (52) by P_t we get

$$w_t = r_{x,t}\alpha_{F,t-1} + r_{H,t}w_{t-1} - C_t + q_t + g(I_t) \tag{53}$$

Where w_t is the real net wealth, $r_{x,t} = (r_{F,t} - r_{H,t})$ is the real excess returns of the F equity over the H equity in which $r_{H,t}$ and $r_{F,t}$ are the real counterparts of $R_{H,t}$, $R_{F,t}$ and where $\alpha_{j,t}$ and q_t are the real counterparts of $\alpha^{nm}_{j,t}$ and Q_t.

B Log Linearized System

The next set of equations summarizes the steady state relations

$$\Upsilon_{ss} = \tau_{ss} = 1 \rightarrow P_{ss} = P_{H,ss} = P$$

$$A_{ss} = A_{ss}^* = 1 \rightarrow Y_{H,ss} = (K_{ss})^\sigma \,; \; Y_{H,ss}^* = (K_{ss}^*)^\sigma$$

$$C_{H,ss} = \lambda C_{ss} \quad C_{F,ss} = (1-\lambda)C_{ss}$$

$$C_{H,ss}^* = (1-\lambda)C_{ss}^* \quad C_{F,ss}^* = \lambda C_{ss}^*$$

$$nx_{ss} = ca_{ss} = w_{ss} = w_{ss}^* = 0$$

$$C_{ss} = C_{H,ss} + C_{H,ss}^* = C_{F,ss}^* + C_{F,ss} = C_{ss}^*$$

$$I_{ss} = \delta K_{ss} \rightarrow C_{ss} = Y_{H,ss} - \delta \left(Y_{H,ss}\right)^{\frac{1}{\sigma}} = (K_{ss})^\sigma - \delta K_{ss}$$

$$C_{ss} = C_{ss}^* \quad \rightarrow \quad Y_{H,ss} = Y_{F,ss}^*$$

$$m_{ss} = \beta_{ss} \rightarrow r_{H,ss} = r_{F,ss} = \frac{1}{\beta_{ss}}$$

$$\frac{1}{\beta_{ss}} = \sigma \left(K_{ss}\right)^{\sigma-1} + 1 - \delta \quad q_{ss} = Y_{H,ss} - \delta K_{ss} = C_{ss}$$

The model equations are (in Gensys format):

$$\hat{a}_t = \rho_A \hat{a}_{t-1} + v\varepsilon_{A,t} + v_{A^*}\varepsilon_{A^*,t} \tag{1.1}$$

$$\hat{a}_t^* = \rho_{A^*} \hat{a}_{t-1}^* + v\varepsilon_{A^*,t} + v_A\varepsilon_{A,t} \tag{1.2}$$

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t} \tag{1.3}$$

$$\hat{\zeta}_t^* = \rho_\zeta \hat{\zeta}_{t-1}^* + \varepsilon_{\zeta^*,t} \tag{1.4}$$

$$\left(1-\psi_1\right)\hat{c}_t - E_t\hat{c}_{t+1} + \beta_{ss}E_t\hat{z}_{H,t+1} + (1-\beta_{ss})E_t\hat{q}_{t+1} - \hat{z}_{H,t} + \hat{\zeta}_t = 0 \tag{1.5}$$

$$\left(1-\psi_1\right)\hat{c}_t^* + (2\lambda-1)\hat{\tau}_t - E_t\hat{c}_{t+1}^* - (2\lambda-1)E_t\hat{\tau}_{t+1} + \beta_{ss}E_t\hat{z}_{H,t+1} \tag{1.6}$$

$$+(1-\beta_{ss})E_t\hat{q}_{t+1} - \hat{z}_{H,t} + \hat{\zeta}_t^* = 0$$

$$\beta_{ss}E\hat{z}^*_{F,t+1} + (1-\beta_{ss})E_t\hat{q}^*_{t+1} + (2\lambda-1)\,E_t\hat{\tau}_{t+1} - \beta_{ss}E_t\hat{z}_{H,t+1} - (1-\beta_{ss})E_t\hat{q}_{t+1} \qquad (1.7)$$

$$= \hat{z}^*_{F,t} + (2\lambda-1)\,\hat{\tau}_t - \hat{z}_{H,t}$$

$$\hat{r}_{x,t} - \beta_{ss}\hat{z}^*_{F,t} - (1-\beta_{ss})\hat{q}^*_t - (2\lambda-1)\,\hat{\tau}_t + \beta_{ss}\hat{z}_{H,t} + (1-\beta_{ss})\hat{q}_t \qquad (1.8)$$

$$= \hat{z}_{H,t-1} - \hat{z}^*_{F,t-1} - (2\lambda-1)\,\hat{\tau}_{t-1}$$

$$\psi_c\hat{q}_t = \hat{y}_{H,t} - \psi_i\hat{\imath}_t - (1-\lambda)\,\psi_c\hat{\tau}_t \qquad (1.9)$$

$$\psi_c\hat{q}^*_t = \hat{y}^*_{F,t} - \psi_i\hat{\imath}^*_t + (1-\lambda)\,\psi_c\hat{\tau}_t \qquad (1.10)$$

$$(1-\psi_1)\,\hat{c}_t + \hat{\zeta}_t - E_t\hat{c}_{t+1} - (1-\lambda)\,[E_t\hat{\tau}_{t+1} - \hat{\tau}_t] - \phi I_{ss}\hat{\imath}_t \qquad (1.11)$$

$$+\,[1-\beta_{ss}\,(1-\delta)]\left[E_t\hat{a}_{t+1} + (\sigma-1)\,E_t\hat{k}_{t+1}\right] + \beta_{ss}\,(1-\delta)\,\phi I_{ss}E_t\hat{\imath}_{t+1} = 0$$

$$(1-\psi_1)\,\hat{c}^*_t + \hat{\zeta}^*_t - E_t\hat{c}^*_{t+1} + (1-\lambda)\,[E_t\hat{\tau}_{t+1} - \hat{\tau}_t] - \phi I_{ss}\hat{\imath}^*_t \qquad (1.12)$$

$$+\,[1-\beta_{ss}\,(1-\delta)]\left[E_t\hat{a}^*_{t+1} + (\sigma-1)\,E_t\hat{k}^*_{t+1}\right] + \beta_{ss}\,(1-\delta)\,\phi I_{ss}E_t\hat{\imath}^*_{t+1} = 0$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{\imath}_{t-1} \qquad (1.13)$$

$$\hat{k}^*_t = (1-\delta)\hat{k}^*_{t-1} + \delta\hat{\imath}^*_{t-1} \qquad (1.14)$$

$$\hat{y}_{H,t} - \hat{a}_t - \sigma\hat{k}_t = 0 \qquad (1.15)$$

$$\hat{y}_{F,t}^* - \hat{a}_t^* - \sigma \hat{k}_t^* = 0 \tag{1.16}$$

$$\hat{w}_t + \psi_c \hat{c}_t - \hat{y}_{H,t} + \psi_i \hat{\imath}_t + (1 - \lambda)(1 - \psi_i)\hat{\tau}_t = \frac{1}{\beta_{ss}}\hat{w}_{t-1} + \varepsilon_t \tag{1.17}$$

$$\hat{y}_{H,t} - \psi_c\left[\lambda \hat{c}_t + (1 - \lambda)\,\hat{c}_t^*\right] - \psi_i \hat{\imath}_t - 2\lambda\,(1 - \lambda)\,\psi_c \theta \hat{\tau}_t = 0 \tag{1.18}$$

$$\hat{y}_{F,t}^* - \psi_c\left[\lambda \hat{c}_t^* + (1 - \lambda)\,\hat{c}_t\right] - \psi_i \hat{\imath}_t^* + 2\lambda\,(1 - \lambda)\,\psi_c \theta \hat{\tau}_t = 0 \tag{1.19}$$

C   The second order

To study the dynamics of the portfolio allocations is necessary a second order approximation of the macro side of the model (besides the third order of the portfolio choice conditions). Since the Devereux and Sutherland procedure delivers a solution for the portfolio allocation dynamics in function only of the first order deviations of the state variables, only the first order solution of the model is used to compute the I/R functions and for simulations. Hence I just summarize here the second order approximation of the model, while the description of the solution conditons and the steps necessary to apply them to this model can be found in the main body of the text. I use the Lombaro and Sutherland (05) second order solution method.

The processes of the shocks are assumed to be linear, so they don't present second order terms

$$\hat{a}_t = \rho_A \hat{a}_{t-1} + v\varepsilon_{A,t} + v_{A^*}\varepsilon_{A^*,t} \tag{2.1}$$

$$\hat{a}_t^* = \rho_{A^*} \hat{a}_{t-1}^* + v\varepsilon_{A^*,t} + v_A \varepsilon_{A,t} \tag{2.2}$$

$$\hat{\zeta}_t = \rho_\zeta \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t} \tag{2.3}$$

$$\hat{\zeta}_t^* = \rho_\zeta \hat{\zeta}_{t-1}^* + \varepsilon_{\zeta^*,t} \tag{2.4}$$

The Euler's equations are just an extension of the first order approximation; but, since the returns are explicitly left in the expressions for simplicity, we also need two equations defining them

$$\hat{\zeta}_t + (1 - \psi_1)\,\hat{c}_t + E_t\hat{r}_{H,t+1} - E_t\hat{c}_{t+1} - \frac{1}{2}\left[\zeta_t^2 + (1 - 2\psi_1)\,\hat{c}_t^2 - E_t\hat{r}_{H,t+1}^2 - E_t\hat{c}_{t+1}^2\right]$$

$$- (1 - \psi_1)\,\hat{\zeta}_t\hat{c}_t - E_t\hat{r}_{H,t+1}\hat{c}_{t+1} = 0 \tag{2.5}$$

$$\hat{\zeta}_t^* + (1 - \psi_1)\,\hat{c}_t^* + (2\lambda - 1)\,\hat{\tau}_t + E_t\hat{r}_{H,t+1} - E_t\hat{c}_{t+1}^* - (2\lambda - 1)\,E_t\hat{\tau}_{t+1} \tag{2.6}$$

$$- \frac{1}{2}\left[\zeta_t^{*2} + (1 - 2\psi_1)\,\hat{c}_t^{*2} + (2\lambda - 1)^2\,\hat{\tau}_t^2 - E_t\hat{r}_{H,t+1}^2 - E_t\hat{c}_{t+1}^{*2} - (2\lambda - 1)^2\,E_t\hat{\tau}_{t+1}\right]$$

$$- (2\lambda - 1)\,\hat{\zeta}_t^*\hat{\tau}_t - (1 - \psi_1)\,\hat{\zeta}_t^*\hat{c}_t^* - (1 - \psi_1)\,(2\lambda - 1)\,\hat{c}_t^*\hat{\tau}_t + (2\lambda - 1)\,E_t\hat{c}_{t+1}^*\hat{\tau}_{t+1}$$

$$- (2\lambda - 1)\,E_t\hat{r}_{H,t+1}\hat{\tau}_{t+1} - E_t\hat{r}_{H,t+1}\hat{c}_{t+1}^* = 0$$

$$\hat{r}_{H,t} = \beta_{ss}\hat{z}_{H,t} + (1 - \beta_{ss})\,\hat{q}_t - \hat{z}_{H,t-1} + \frac{1}{2}\left[\beta_{ss}\hat{z}_{H,t}^2 + (1 - \beta_{ss})\,\hat{q}_t^2 + \hat{z}_{H,t-1}^2 - \hat{r}_{H,t}^2\right]$$

$$- \beta_{ss}\hat{z}_{H,t-1}\hat{z}_{H,t} - (1 - \beta_{ss})\,\hat{q}_t\hat{z}_{H,t-1} \tag{2.7}$$

$$\hat{r}_{F,t} = \beta_{ss}\hat{z}_{F,t}^* + (1 - \beta_{ss})\,\hat{q}_t^* - \hat{z}_{F,t-1}^* + (2\lambda - 1)\,\hat{\tau}_t - (2\lambda - 1)\,\hat{\tau}_{t-1}$$

$$+ \frac{1}{2}\left[\beta_{ss}\hat{z}_{F,t}^{*2} + (1 - \beta_{ss})\,\hat{q}_t^{*2} + \hat{z}_{F,t-1}^{*2} + (2\lambda - 1)^2\,\hat{\tau}_t^2 + (2\lambda - 1)^2\,\hat{\tau}_{t-1}^2 - \hat{r}_{F,t}^2\right]$$

$$+ \beta_{ss}\,(2\lambda - 1)\,\hat{z}_{F,t}^*\hat{\tau}_t - \beta_{ss}\,(2\lambda - 1)\,\hat{z}_{F,t}^*\hat{\tau}_{t-1} - \beta_{ss}\hat{z}_{F,t-1}^*\hat{z}_{F,t}^* - (1 - \beta_{ss})\,(2\lambda - 1)\,\hat{q}_t^*\hat{\tau}_t$$

$$- (1 - \beta_{ss})\,(2\lambda - 1)\,\hat{q}_t^*\hat{\tau}_{t-1} - (1 - \beta_{ss})\,\hat{q}_t^*\hat{z}_{F,t-1}^* + (2\lambda - 1)\,\hat{z}_{F,t-1}^*\hat{\tau}_t$$

$$+ (2\lambda - 1)\,\hat{z}_{F,t-1}^*\hat{\tau}_{t-1} - (2\lambda - 1)^2\,\hat{\tau}_t\hat{\tau}_{t-1} \tag{2.8}$$

Equation (2.7) and (2.8) can be used in the excess returns condition derived from the portfolio optimality conditions up to the second order approximation. The definition of the first order term of the approximated excess returns can be introduced in the system in order to make its solution deliver the terms we need to apply the next step of the Devereux and Sutherland solution procedure. Even though a variable $r_x = r_F - r_H$ is defined in the system as excess return, we just use in the solution method a definition $\hat{r}_x = \hat{r}_F - \hat{r}_H$ of the approximated excess return which has a meaning only at the first order, $\hat{r}_x$ doesn't have an explicit second order derivation from the corresponding level variable

$$\hat{r}_{x,t} = \hat{r}_{F,t} - \hat{r}_{H,t} \tag{2.9}$$

$$E_t \left(\hat{r}_{F,t+1} - \hat{r}_{H,t+1} \right) = E_t \left[-\frac{1}{2} \left(\hat{r}_{F,t+1}^2 - \hat{r}_{H,t+1}^2 \right) + \frac{1}{2} \left(\hat{c}_{t+1} + \hat{c}_{t+1}^* + (2\lambda - 1) \hat{\tau}_{t+1} \right) \left(\hat{r}_{F,t+1} - \hat{r}_{Ht+1} \right) \right] \tag{2.10}$$

or

$$E_t \left(\hat{r}_{x,t+1} \right) = E_t \left[-\frac{1}{2} \left(\hat{r}_{F,t+1}^2 - \hat{r}_{H,t+1}^2 \right) + \frac{1}{2} \left(\hat{c}_{t+1} + \hat{c}_{t+1}^* + (2\lambda - 1) \hat{\tau}_{t+1} \right) \left(\hat{r}_{x,t+1} \right) \right]$$

The profits function, the intertemporal cash flow maximization condition, the capital accumulation and the production function are standard

$$\psi_c \hat{q}_t = \hat{y}_{H,t} - (1-\lambda) \psi_c \hat{\tau}_t - \psi_i \hat{\imath}_t + \frac{1}{2} [\hat{y}_{H,t}^2 + (1-\lambda)(1 + \lambda\theta - 2\lambda) \psi_c \hat{\tau}_t^2$$

$$-\psi_i (1 + \phi I_{ss}) \hat{\imath}_t^2 - \psi_c \hat{q}_t^2] - (1-\lambda) \hat{y}_{H,t} \hat{\tau}_t + (1-\lambda) \psi_i \hat{\imath}_t \hat{\tau}_t \tag{2.11}$$

$$\psi_c \hat{q}_t^* = \hat{y}_{F,t}^* + (1-\lambda) \psi_c \hat{\tau}_t - \psi_i \hat{\imath}_t^* + \frac{1}{2} [\hat{y}_{F,t}^{*2} + (1-\lambda)(1 + \lambda\theta - 2\lambda) \psi_c \hat{\tau}_t^2$$

$$-\psi_i (1 + \phi I_{ss}) \hat{\imath}_t^{*2} - \psi_c \hat{q}_t^{*2}] + (1-\lambda) \hat{y}_{F,t}^* \hat{\tau}_t - (1-\lambda) \psi_i \hat{\imath}_t^* \hat{\tau}_t \tag{2.12}$$

$$\hat{\zeta}_t + (1 - \psi_1)\,\hat{c}_t - E_t\hat{c}_{t+1} - (1 - \lambda)\,(E_t\hat{\tau}_{t+1} - \hat{\tau}_t)$$

$$+\frac{1}{2}\left[(1-\lambda)(1+\lambda\theta-2\lambda)\left(E_t\hat{\tau}_{t+1}^2 - \hat{\tau}_t^2\right) - \zeta_t^2 - (1-2\psi_1)\,\hat{c}_t^2 + E_t\hat{c}_{t+1}^2\right]$$

$$-(1-\lambda)\,\hat{\zeta}_t\hat{\tau}_t - (1-\lambda)(1-\psi_1)\,\hat{\tau}_t\hat{c}_t - (1-\psi_1)\,\hat{\zeta}_t\hat{c}_t + (1-\lambda)\,E_t\hat{\tau}_{t+1}\hat{c}_{t+1}$$

$$-\phi I_{ss}\left[\hat{\imath}_t + \frac{1}{2}\phi\hat{\imath}_t^2 - \hat{\imath}_t\hat{\zeta}_t - (1-\psi_1)\,\hat{c}_t\hat{\imath}_t\right]$$

$$+\left[1 - \beta_{ss}(1-\delta)\right]\left[E_t\hat{a}_{t+1} + (\sigma-1)\,E_t\hat{k}_{t+1} + \frac{1}{2}\left(E_t\hat{a}_{t+1}^2 + (\sigma-1)^2\,E_t\hat{k}_{t+1}^2\right)\right.$$

$$-E_t\hat{a}_{t+1}\hat{c}_{t+1} + (\sigma-1)\,E_t\hat{a}_{t+1}\hat{k}_{t+1} - (\sigma-1)\,E_t\hat{k}_{t+1}\hat{c}_{t+1}$$

$$\left.-(1-\lambda)\,E_t\hat{a}_{t+1}\hat{\tau}_{t+1} - (1-\lambda)(\sigma-1)\,E_t\hat{k}_{t+1}\hat{\tau}_{t+1}\right]$$

$$+\beta_{ss}(1-\delta)\phi I_{ss}E_t\left[\hat{\imath}_{t+1} + \frac{1}{2}\hat{\imath}_{t+1}^2 - \hat{\imath}_{t+1}\hat{c}_{t+1}\right] = 0 \qquad (2.13)$$

$$\hat{\zeta}_t^* + (1 - \psi_1)\,\hat{c}_t^* - E_t\hat{c}_{t+1}^* + (1 - \lambda)\,(E_t\hat{\tau}_{t+1} - \hat{\tau}_t)$$

$$+\frac{1}{2}\left[(1-\lambda)(1+\lambda\theta-2\lambda)\left(E_t\hat{\tau}_{t+1}^2 - \hat{\tau}_t^2\right) - \zeta_t^{*2} - (1-2\psi_1)\,\hat{c}_t^{*2} + E_t\hat{c}_{t+1}^{*2}\right]$$

$$+(1-\lambda)\,\hat{\zeta}_t^*\hat{\tau}_t + (1-\lambda)(1-\psi_1)\,\hat{\tau}_t\hat{c}_t^* - (1-\psi_1)\,\hat{\zeta}_t^*\hat{c}_t^* - (1-\lambda)\,E_t\hat{\tau}_{t+1}\hat{c}_{t+1}^*$$

$$-\phi I_{ss}\left[\hat{\imath}_t^* + \frac{1}{2}\phi\hat{\imath}_t^{*2} - \hat{\imath}_t^*\hat{\zeta}_t^* - (1-\psi_1)\,\hat{c}_t^*\hat{\imath}_t^*\right]$$

$$+\left[1 - \beta_{ss}(1-\delta)\right]\left[E_t\hat{a}_{t+1}^* + (\sigma-1)\,E_t\hat{k}_{t+1}^* + \frac{1}{2}\left(E_t\hat{a}_{t+1}^{*2} + (\sigma-1)^2\,E_t\hat{k}_{t+1}^{*2}\right)\right.$$

$$-E_t\hat{a}_{t+1}^*\hat{c}_{t+1}^* + (\sigma-1)\,E_t\hat{a}_{t+1}^*\hat{k}_{t+1}^* - (\sigma-1)\,E_t\hat{k}_{t+1}^*\hat{c}_{t+1}^*$$

$$\left.+(1-\lambda)\,E_t\hat{a}_{t+1}^*\hat{\tau}_{t+1} + (1-\lambda)(\sigma-1)\,E_t\hat{k}_{t+1}^*\hat{\tau}_{t+1}\right]$$

$$+\beta_{ss}(1-\delta)\phi I_{ss}E_t\left[\hat{\imath}_{t+1}^* + \frac{1}{2}\hat{\imath}_{t+1}^{*2} - \hat{\imath}_{t+1}^*\hat{c}_{t+1}^*\right] = 0 \qquad (2.14)$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{\imath}_{t-1} + \frac{1}{2}\left[(1-\delta)\,\hat{k}_{t-1}^2 + \delta\hat{\imath}_{t-1}^2 - \hat{k}_t^2\right] \qquad (2.15)$$

$$\hat{k}_t^* = (1-\delta)\hat{k}_{t-1}^* + \delta\hat{\imath}_{t-1}^* + \frac{1}{2}\left[(1-\delta)\,\hat{k}_{t-1}^{*2} + \delta\hat{\imath}_{t-1}^{*2} - \hat{k}_t^{*2}\right] \qquad (2.16)$$

$$\hat{y}_{H,t} = \hat{a}_t + \sigma\hat{k}_t + \frac{1}{2}\left[\hat{a}_t^2 + \sigma^2\hat{k}_t^2 - \hat{y}_{H,t}^2\right] + \sigma\hat{k}_t\hat{a}_t \qquad (2.17)$$

$$\hat{y}_{F,t}^* = \hat{a}_t^* + \sigma \hat{k}_t^* + \frac{1}{2} \left[\hat{a}_t^{*2} + \sigma^2 \hat{k}_t^{*2} - \hat{y}_{F,t}^{*2} \right] + \sigma \hat{k}_t^* \hat{a}_t^* \tag{2.18}$$

Finally the last three equations are the wealth equation or H-country budget constraint and the two market clearing conditions

$$\hat{w}_t = \tilde{\alpha}_{F,ss} \hat{r}_{x,t} + \hat{y}_{H,t} - \psi_c \hat{c}_t - (1-\lambda)(1-\psi_i)\hat{\tau}_t - \psi_i \hat{\imath}_t + \frac{1}{\beta_{ss}} \hat{w}_{t-1}$$

$$+ \frac{1}{2} \left[\tilde{\alpha}_{F,ss} \left(\hat{r}_{F,t}^2 - \hat{r}_{H,t}^2 \right) + \hat{y}_{H,t}^2 - \psi_c \hat{c}_t^2 + (1-\lambda)(1-\psi_i)(1+\lambda\theta - 2\lambda)\hat{\tau}_t^2 - \psi_i \hat{\imath}_t^2 \right]$$

$$- (1-\lambda)\hat{\tau}_t \hat{y}_{H,t} + (1-\lambda)\psi_i \hat{\tau}_t \hat{\imath}_t + \frac{1}{\beta_{ss}} \hat{w}_{t-1} \hat{r}_{H,t} + \varepsilon_t \tag{2.19}$$

where $\varepsilon_t = \hat{\alpha}_{F,t-1} \hat{r}_{x,t}$

$$\hat{y}_{H,t} = \lambda \psi_c \hat{c}_t + (1-\lambda)\psi_c \hat{c}_t^* + \psi_i \hat{\imath}_t + 2(1-\lambda)\psi_c \lambda\theta \hat{\tau}_t$$

$$+ \frac{1}{2} \left[\lambda \psi_c \hat{c}_t^2 + (1-\lambda)\psi_c \hat{c}_t^{*2} + \psi_i \hat{\imath}_t^2 + (1-\lambda)(4\lambda - 3)\psi_c \lambda\theta \hat{\tau}_t^2 - \hat{y}_{H,t}^2 \right]$$

$$+ (1-\lambda)\hat{y}_{H,t}\hat{\tau}_t - (1-\lambda)\psi_i \hat{\imath}_t \hat{\tau}_t + (1-\lambda)(\lambda + \lambda\theta - 1)\psi_c \hat{c}_t^* \hat{\tau}_t - \lambda(1-\lambda)(1-\theta)\psi_c \hat{c}_t \hat{\tau}_t \tag{2.20}$$

$$\hat{y}_{F,t}^* = \lambda \psi_c \hat{c}_t^* + (1-\lambda)\psi_c \hat{c}_t + \psi_i \hat{\imath}_t^* - 2(1-\lambda)\psi_c \lambda\theta \hat{\tau}_t$$

$$+ \frac{1}{2} \left[\lambda \psi_c \hat{c}_t^{*2} + (1-\lambda)\psi_c \hat{c}_t^2 + \psi_i \hat{\imath}_t^{*2} + (1-\lambda)(4\lambda - 3)\psi_c \lambda\theta \hat{\tau}_t^2 - \hat{y}_{F,t}^{*2} \right]$$

$$- (1-\lambda)\hat{y}_{F,t}^* \hat{\tau}_t + (1-\lambda)\psi_i \hat{\imath}_t^* \hat{\tau}_t - (1-\lambda)(\lambda + \lambda\theta - 1)\psi_c \hat{c}_t \hat{\tau}_t + \lambda(1-\lambda)(1-\theta)\psi_c \hat{c}_t^* \hat{\tau}_t \tag{2.21}$$

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Figures
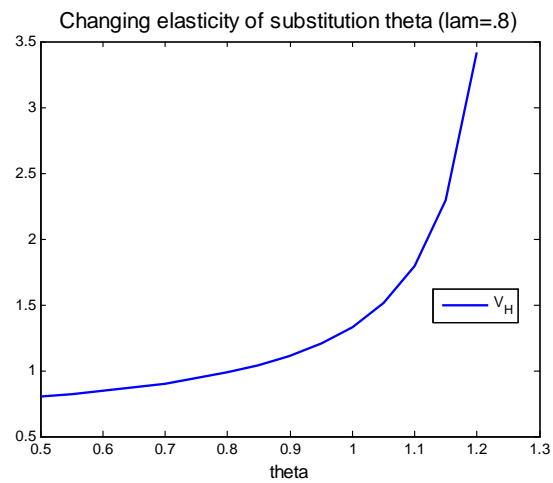
Figure 1: the role of the elasticity of substitution θ
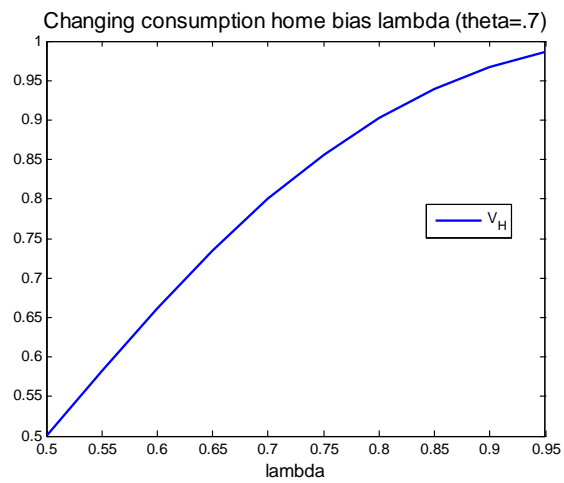
Figure 2: the role of the consumption home bias λ
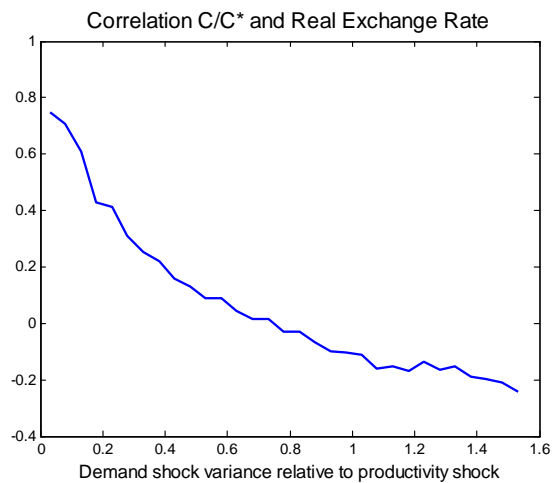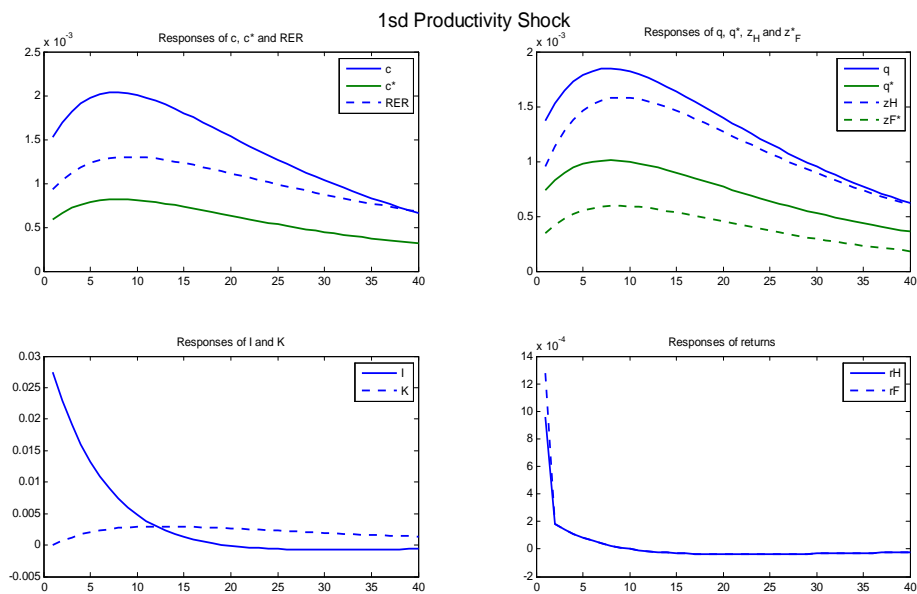
Figure 3: the Backus-Smith puzzle

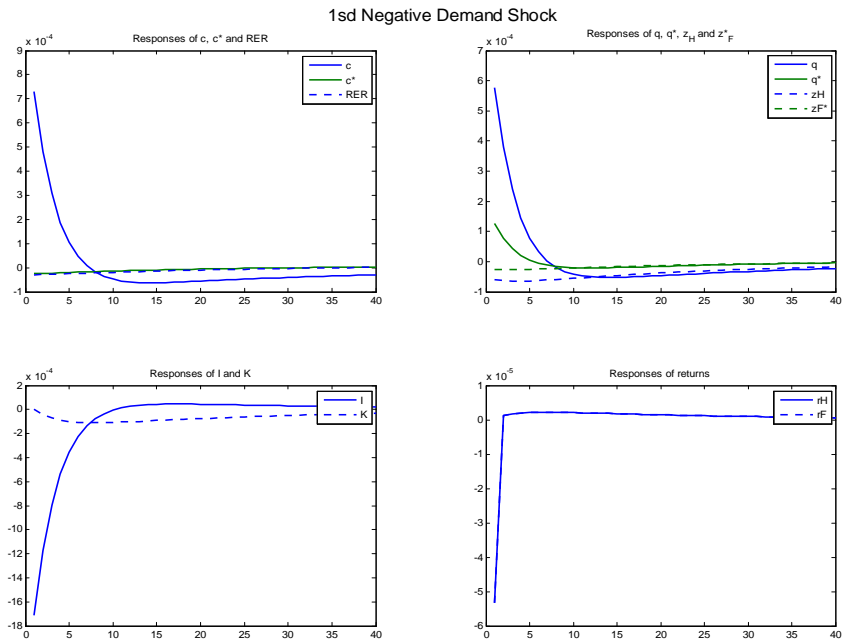Figure 4: IRF to a 1 standard deviation productivity shock

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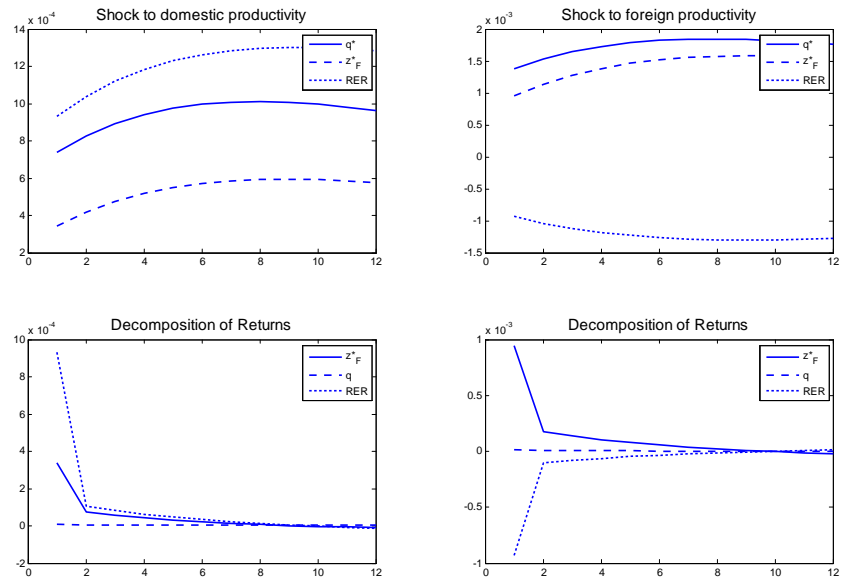Figure 5: IRF to a (negative) 1 standard deviation demand shock

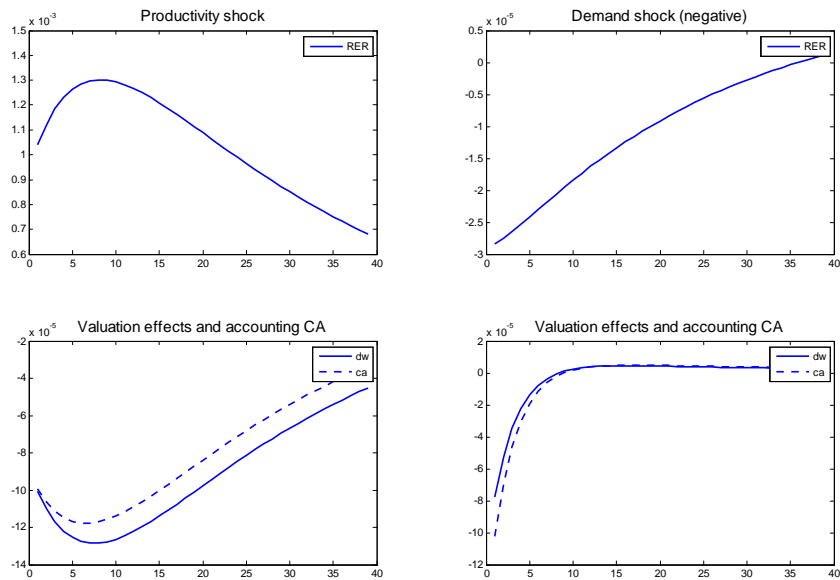Figure 6: Decompositon of the assets returns in response to pruductivity shocks
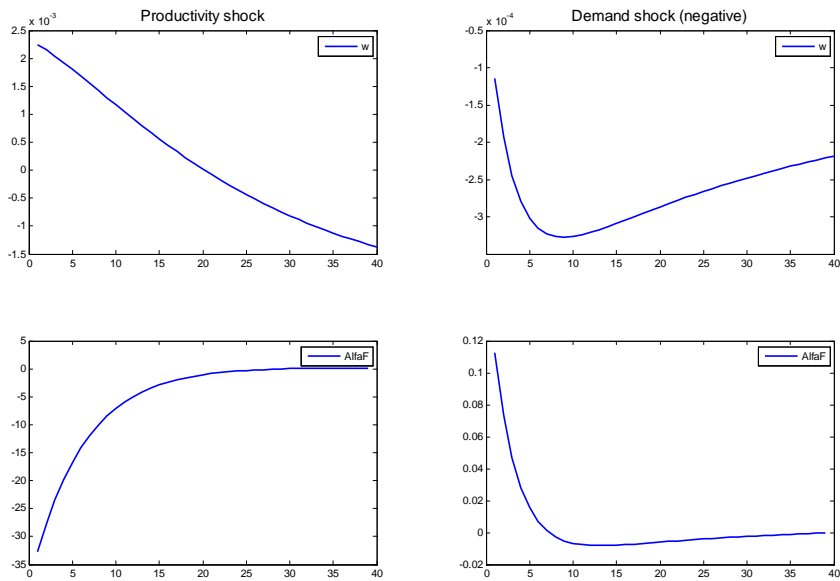
Figure 7: The effect of valuation on the CA

Figure 8: Responses of net wealth $\hat{w}$ and foreign assets $\hat{\alpha}_{F,t}$ held by the home consumer to productivity and demand shocks
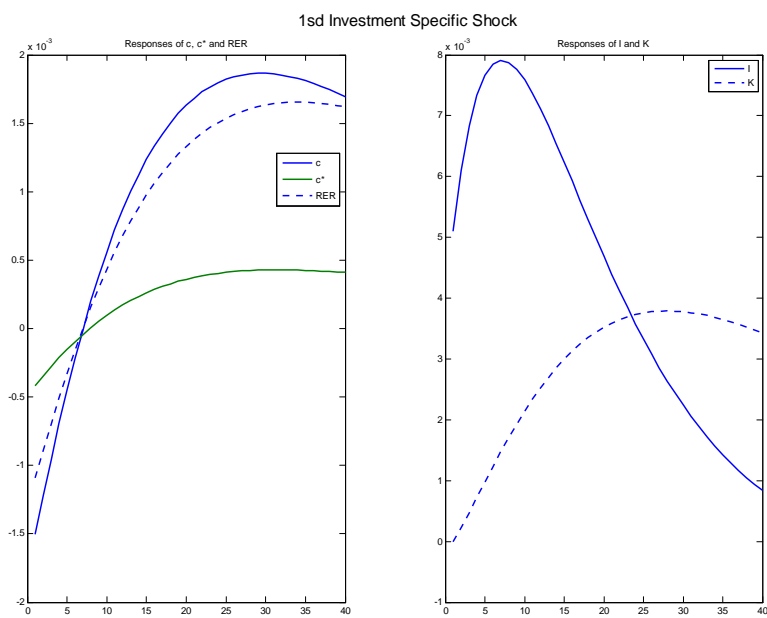
Figure 9: IRF to a 1 standard deviation investment specific shock